Lecture 11

2022/2023

Microwave Devices and Circuits for Radiocommunications

2022/2023

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course or laboratory)**
- Lectures- **associate professor Radu Damian**
  - Tuesday 12-14, ~~Online~~, P8
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 21-28.02.2023 (t2 and t3 not announced, lecture)
    - 3att.=+0.5p
  - **all materials/equipments authorized**

2022/2023

- Laboratory – **associate professor Radu Damian**
  - Tuesday 08-12, II.13 / (08:10)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 21.02.2022)
    - personal homework

Materials

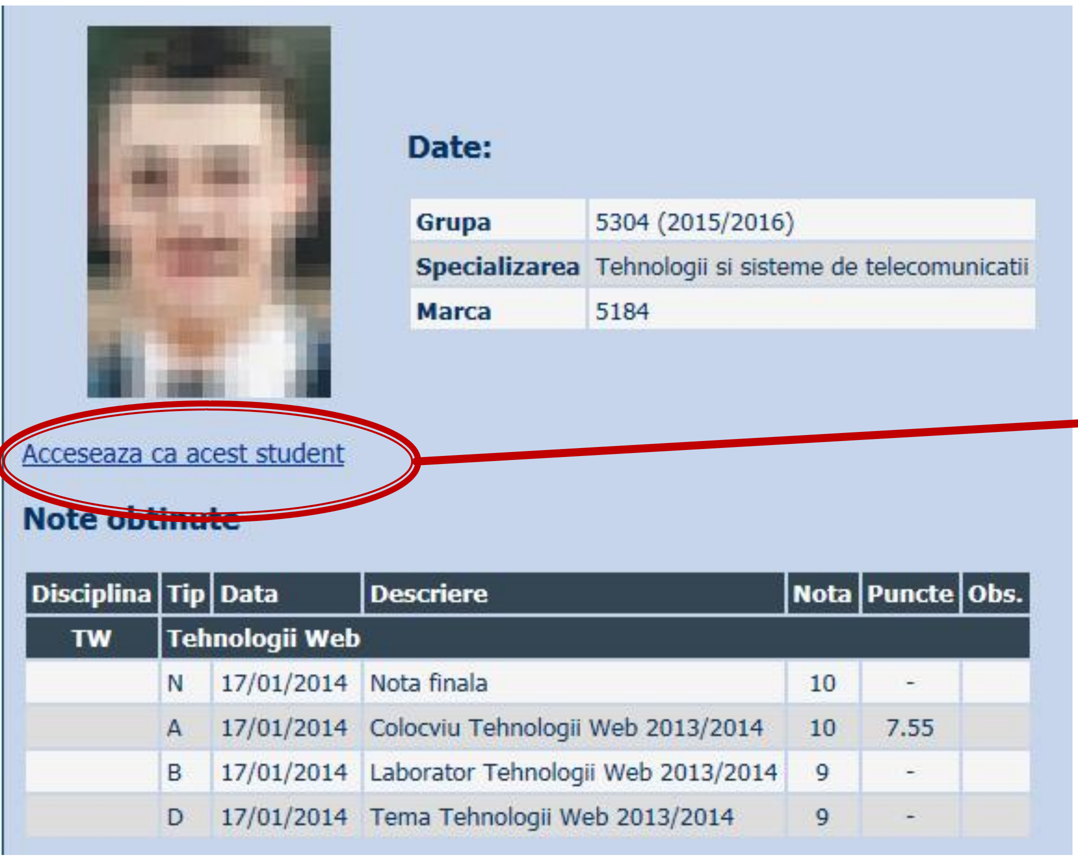
- http://rf-opto.etti.tuiasi.ro

Materials

- RF-OPTO
  - **http://rf-opto.etti.tuiasi.ro**
- **David Pozar, "Microwave Engineering"**, Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by **email**/online exam
  - used at lectures/laboratory

Access

- ~~Not customized~~



Date:

Grupa	5304 (2015/2016)
Specializarea	Tehnologii si sisteme de telecomunicatii
Marca	5184

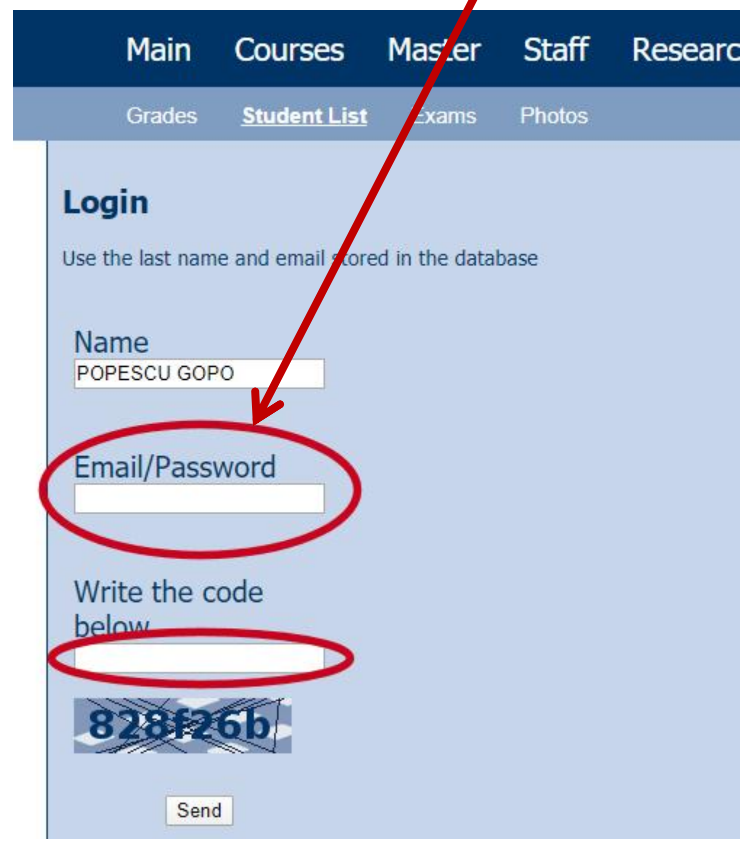
Acceseaza ca acest student

Note obtinute

Disciplina	Tip	Data	Descriere	Nota	Puncte	Obs.
TW	Tehnologii Web					
	N	17/01/2014	Nota finala	10	-	
	A	17/01/2014	Colocviu Tehnologii Web 2013/2014	10	7.55	
	B	17/01/2014	Laborator Tehnologii Web 2013/2014	9	-	
	D	17/01/2014	Tema Tehnologii Web 2013/2014	9	-	

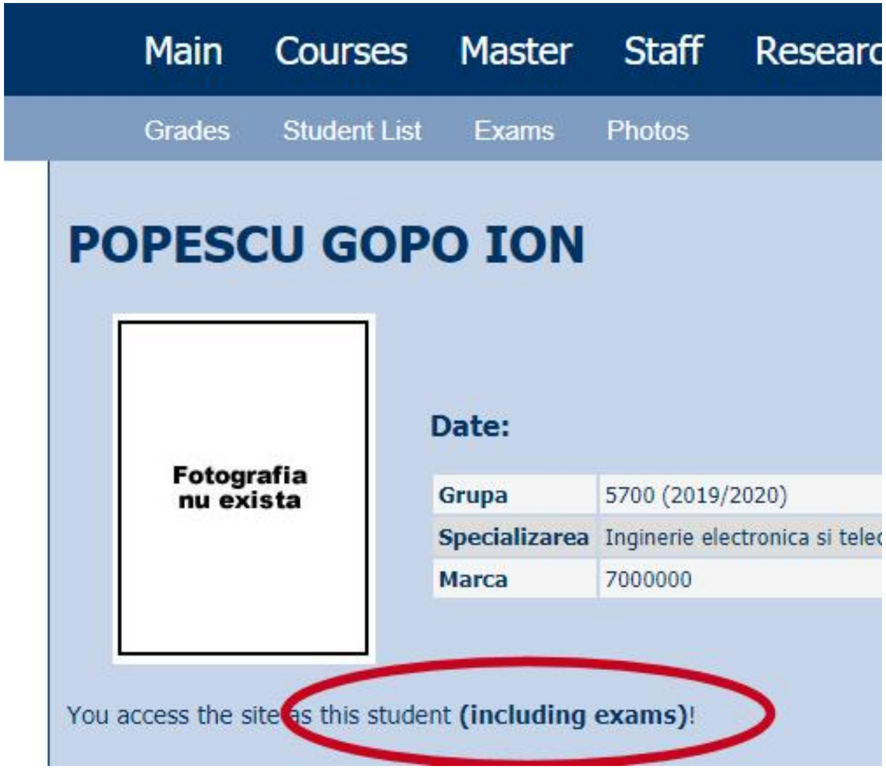
Online

- access to **online exams** requires the **password** received by email



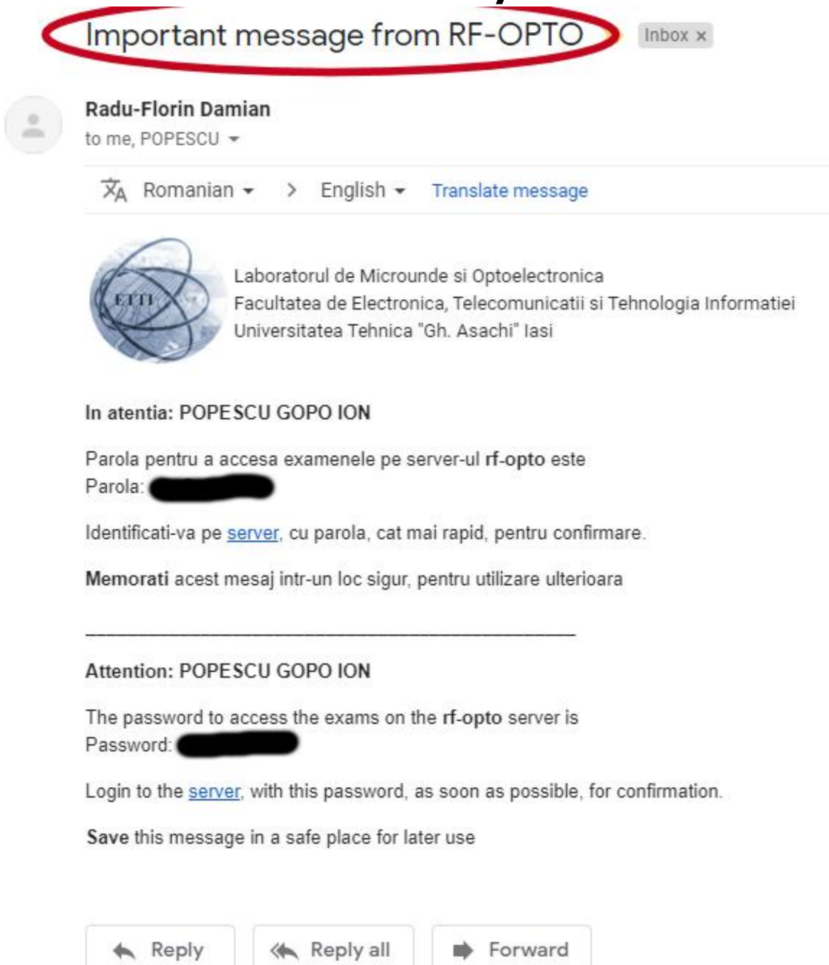
Online

- access email/password



Password

- received by email



Important message from RF-OPTO Inbox ×

Radu-Florin Damian
to me, POPESCU

Romanian > English Translate message

Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
Universitatea Tehnica "Gh. Asachi" Iasi

In atentia: POPESCU GOPO ION

Parola pentru a accesa examenele pe server-ul **rf-opto** este
Parola: [redacted]

Identificati-va pe server, cu parola, cat mai rapid, pentru confirmare.

Memorati acest mesaj intr-un loc sigur, pentru utilizare ulterioara

____

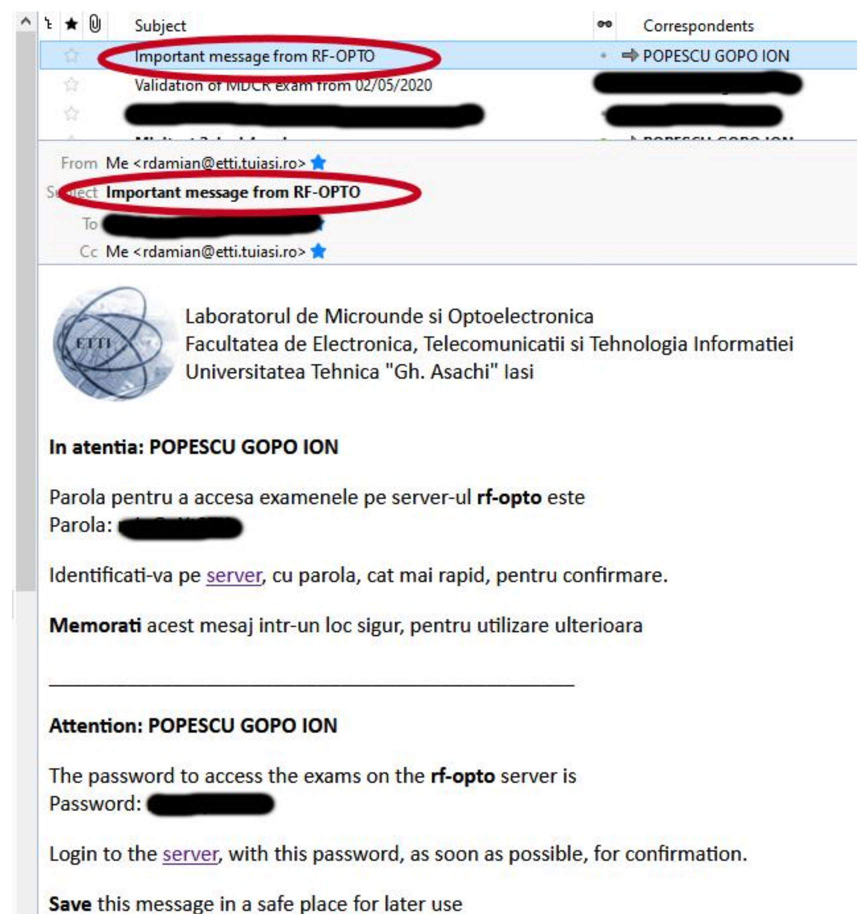
Attention: POPESCU GOPO ION

The password to access the exams on the **rf-opto** server is
Password: [redacted]

Login to the server, with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

Reply | Reply all | Forward



Subject	Correspondents
Important message from RF-OPTO	POPESCU GOPO ION
Validation of MDCR exam from 02/05/2020	[redacted]
[redacted]	[redacted]

From Me <rdamian@etti.tuiasi.ro>
Subject **Important message from RF-OPTO**
To [redacted]
Cc Me <rdamian@etti.tuiasi.ro>

Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
Universitatea Tehnica "Gh. Asachi" Iasi

In atentia: POPESCU GOPO ION

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____

Attention: POPESCU GOPO ION

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Password: [redacted]

Login to the server, with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

Online exam manual

- The online exam app used for:
  - ~~lectures (attendance)~~
  - laboratory
  - project
  - ~~examinations~~

Materials

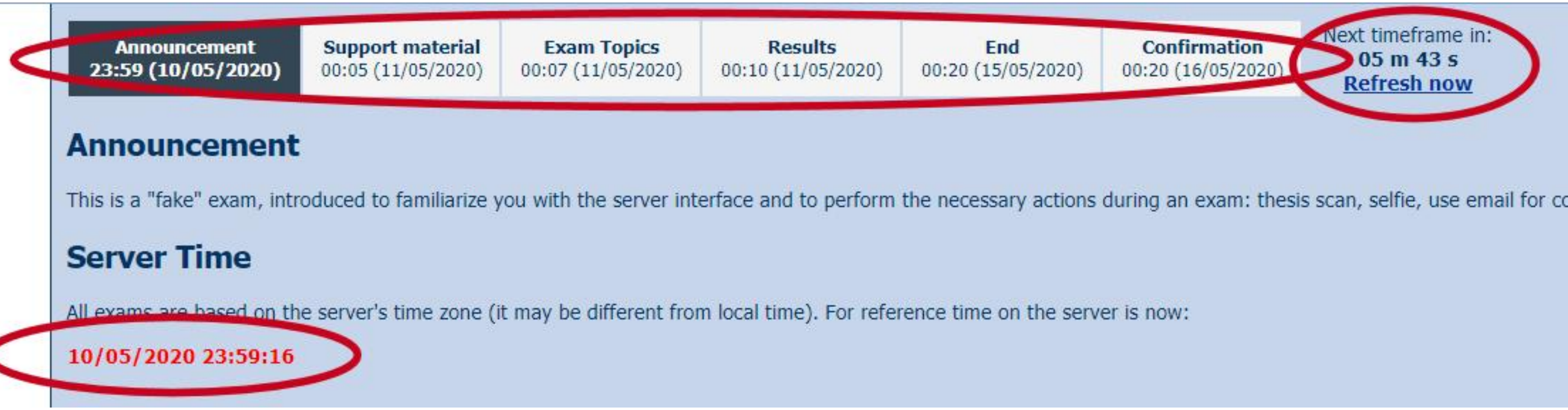
Other data

Manual examen on-line (pdf, 2.65 MB, ro,)
Simulare Examen (video) (mp4, 65.12 MB, ro,)

Microwave Devices and Circuits (Englis

Examen online

- always against a **timetable**
  - long period (lecture attendance/laboratory results)
  - ~~short period (tests: 15min, exam: 2h)~~



Online results submission

- many numerical values/files

Schema finala	Rezultate - castig	Rezultate - zgomot	Fisier justificare calcul (factor andrei)	Fisier zap (optional)	T1, fisier parmetri s	T2, fisier parmetri s	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Ze1	Zo1	Ze2	Zo2	Ze3	Zo3	Ze4	Zo4	Ze5	Zo5	Ze6
86 - 5428 - 259 ...	86 - 5428 - 260 ...	86 - 5428 - 261 ...	86 - 5428 - 316 ...	-	86 - 5428 - 314 ...	86 - 5428 - 315 ...	148.33	155.88	202.12	164.35	180.91	30.29	185.19	79.9	37	68.89	45.14	61.83	45.05	57.97	46.02	61.85	45.05	68.8
86 - 5622 - 259 ...	86 - 5622 - 260 ...	86 - 5622 - 261 ...	86 - 5622 - 316 ...	86 - 5622 - 262 ...	86 - 5622 - 314 ...	86 - 5622 - 315 ...	26.97	153.5	34.64	35.79	55.56	26.212	10.693	0	0	0	0	0	0	0	0	0	0	0
86 - 5488 - 259 ...	86 - 5488 - 260 ...	86 - 5488 - 261 ...	86 - 5488 - 316 ...	86 - 5488 - 262 ...	86 - 5488 - 314 ...	86 - 5488 - 315 ...	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
86 - 5391 - 259 ...	86 - 5391 - 260 ...	86 - 5391 - 261 ...	86 - 5391 - 316 ...	-	-	-	50	50	50	50	50	50	50	70.14	40.39	61.85	44.59	55.7	45.2	54.89	45.38	58.65	45.8	70.0
86 - 5664 - 259 ...	86 - 5664 - 260 ...	86 - 5664 - 261 ...	86 - 5664 - 316 ...	-	86 - 5664 - 314 ...	86 - 5664 - 315 ...	168.02	150.5	178.28	133.75	92.12	121.67	144.48	94.36	36.19	70.77	42.56	65.69	42.05	55.17	42.29	65.59	42.05	70.7
86 - 5665 - 259 ...	86 - 5665 - 260 ...	86 - 5665 - 261 ...	86 - 5665 - 316 ...	-	86 - 5665 - 314 ...	86 - 5665 - 315 ...	162.2	80.8	209.2	140.85	135.1	183.7	167.6	94.58	36.15	78.16	39.77	65.57	45.05	65.57	45.05	78.16	39.77	94.5
86 - 5433 - 259 ...	86 - 5433 - 260 ...	86 - 5433 - 261 ...	86 - 5433 - 316 ...	-	86 - 5433 - 314 ...	86 - 5433 - 315 ...	165.138	106.228	226.157	130.134	72.71	180.177	164.616	101.36	36.11	77.22	42.49	68.02	45.62	60	45.42	68.02	45.62	77.2
86 - 5608 - 259 ...	86 - 5608 - 260 ...	86 - 5608 - 261 ...	86 - 5608 - 316 ...	-	86 - 5608 - 314 ...	86 - 5608 - 315 ...	150.84	152.5	30.94	32.37	54.36	19.837	29.85	64.14	40.145	54.32	46.32	53.8	46.7	53.8	46.7	54.32	46.32	54.9
86 - 5555 - 259 ...	86 - 5555 - 260 ...	86 - 5555 - 261 ...	86 - 5555 - 316 ...	-	86 - 5555 - 314 ...	86 - 5555 - 315 ...	168.001	150.288	178.399	133.115	92.491	121.257	144.126	97.05	36.16	71.13	43.09	65.45	42.12	55.66	42.18	65.45	42.12	71.1

Online results submission

- many numerical values

Z1	Z2	Z3	Z4	Z5	Z6	Z7
148.33	155.88	202.12	164.35	180.91	30.29	185.19
26.97	153.5	34.64	35.79	55.56	26.212	10.693
0	0	0	0	0	0	0
50	50	50	50	50	50	50

Online results submission

Grade = Quality of the work +

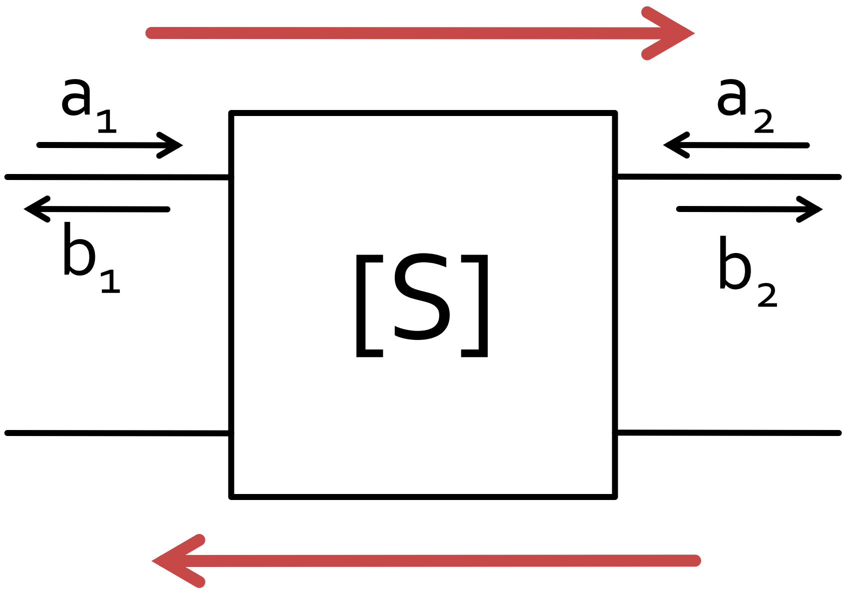
+ Quality of the submission

Recap

General theory

Microwave Network Analysis

Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

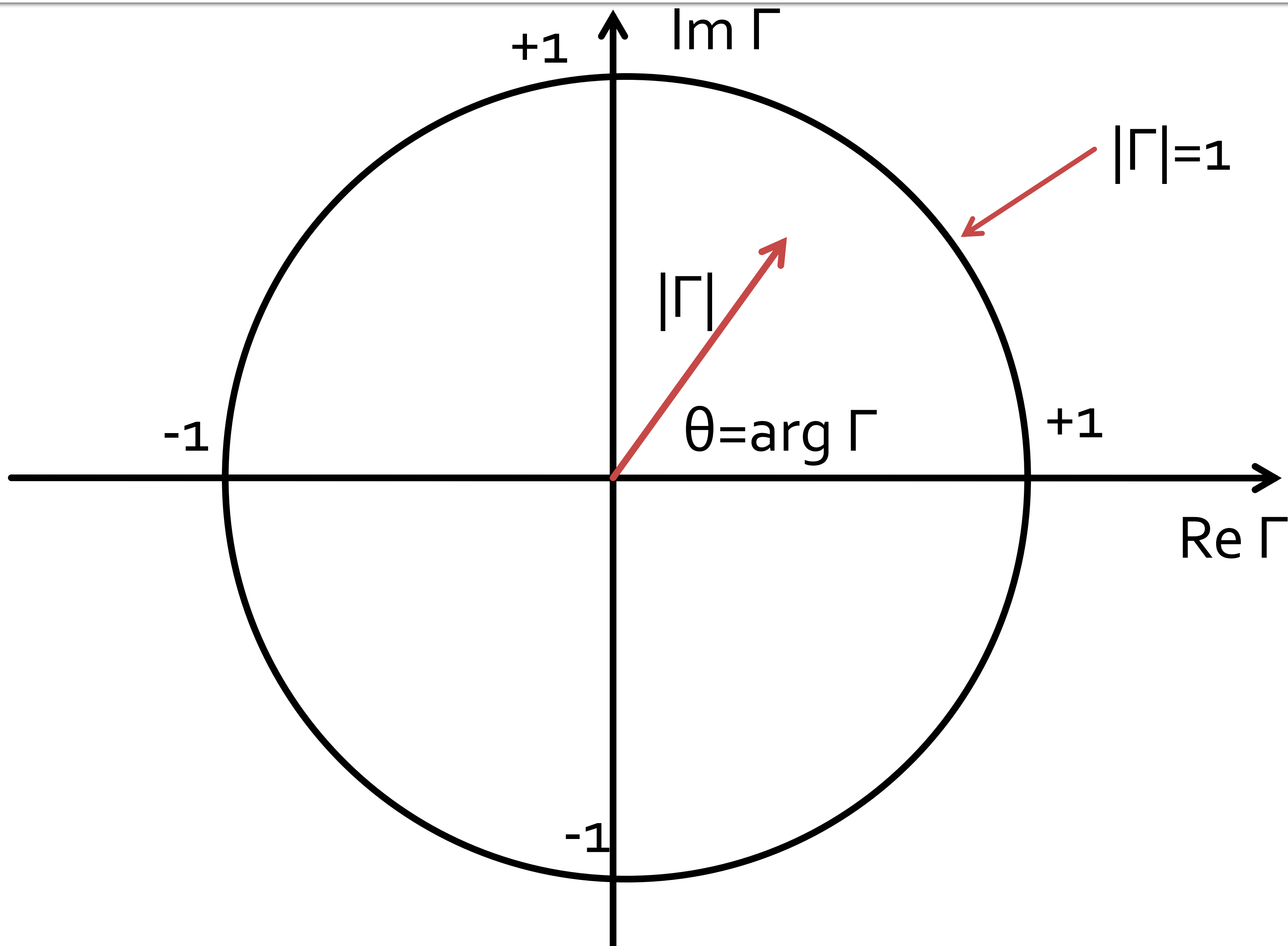
$$|S_{21}|^2 = \frac{Power\ in\ Z_0\ load}{Power\ from\ Z_0\ source}$$

- a,b
  - information about signal power **AND** signal phase
- S_{ij}
  - network effect (gain) over signal power **including** phase information

Impedance Matching

The Smith Chart

The Smith Chart



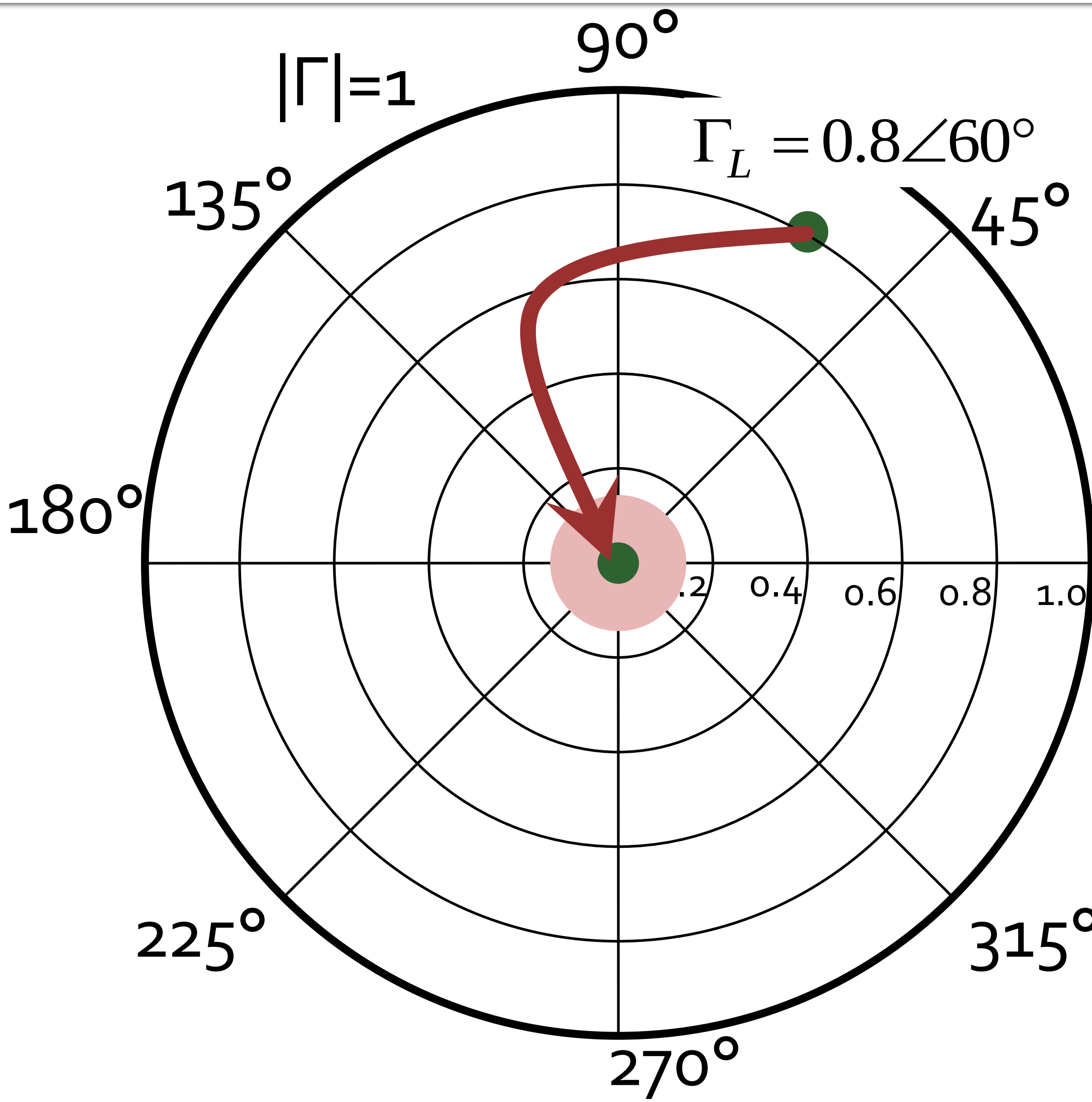
Impedance matching

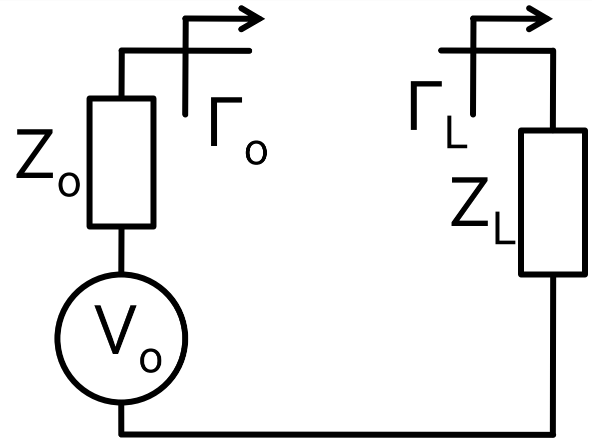
Impedance Matching with lumped elements (L Networks)

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

The Smith Chart, reflection coefficient, impedance matching





Matching Z_L load to Z_o source.
We normalize Z_L over Z_o

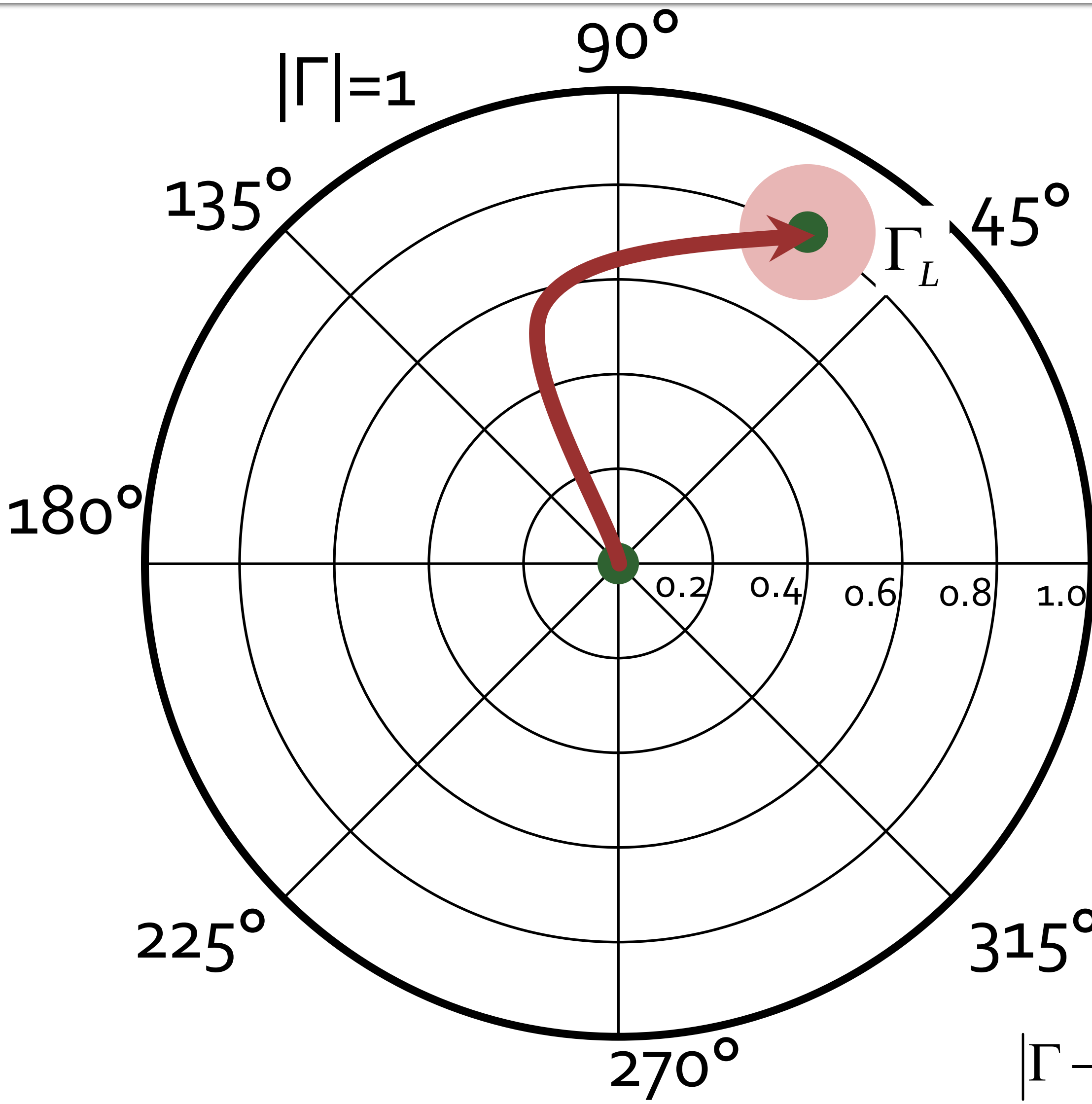
$$Z_L = 21.429\Omega + j \cdot 82.479\Omega$$
$$z_L = 0.429 + j \cdot 1.65$$
$$\Gamma_L = 0.8\angle 60°$$

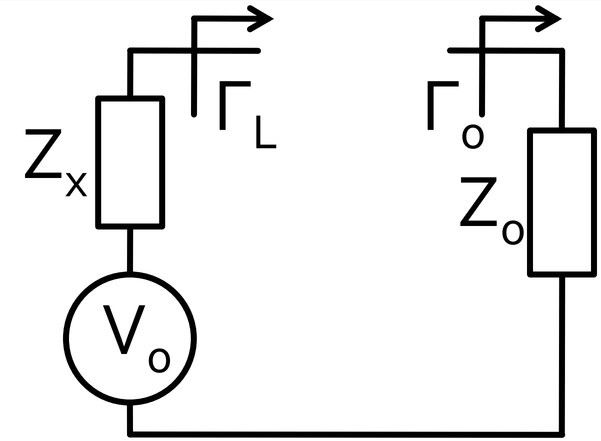
We must move the point denoting the reflection coefficient in the area where with a Z_o source we have:

$\Gamma_0 = 0$ perfect match

$|\Gamma_0| \leq \Gamma_m$ "good enough" match

The Smith Chart, matching, $Z_L=Z_o$





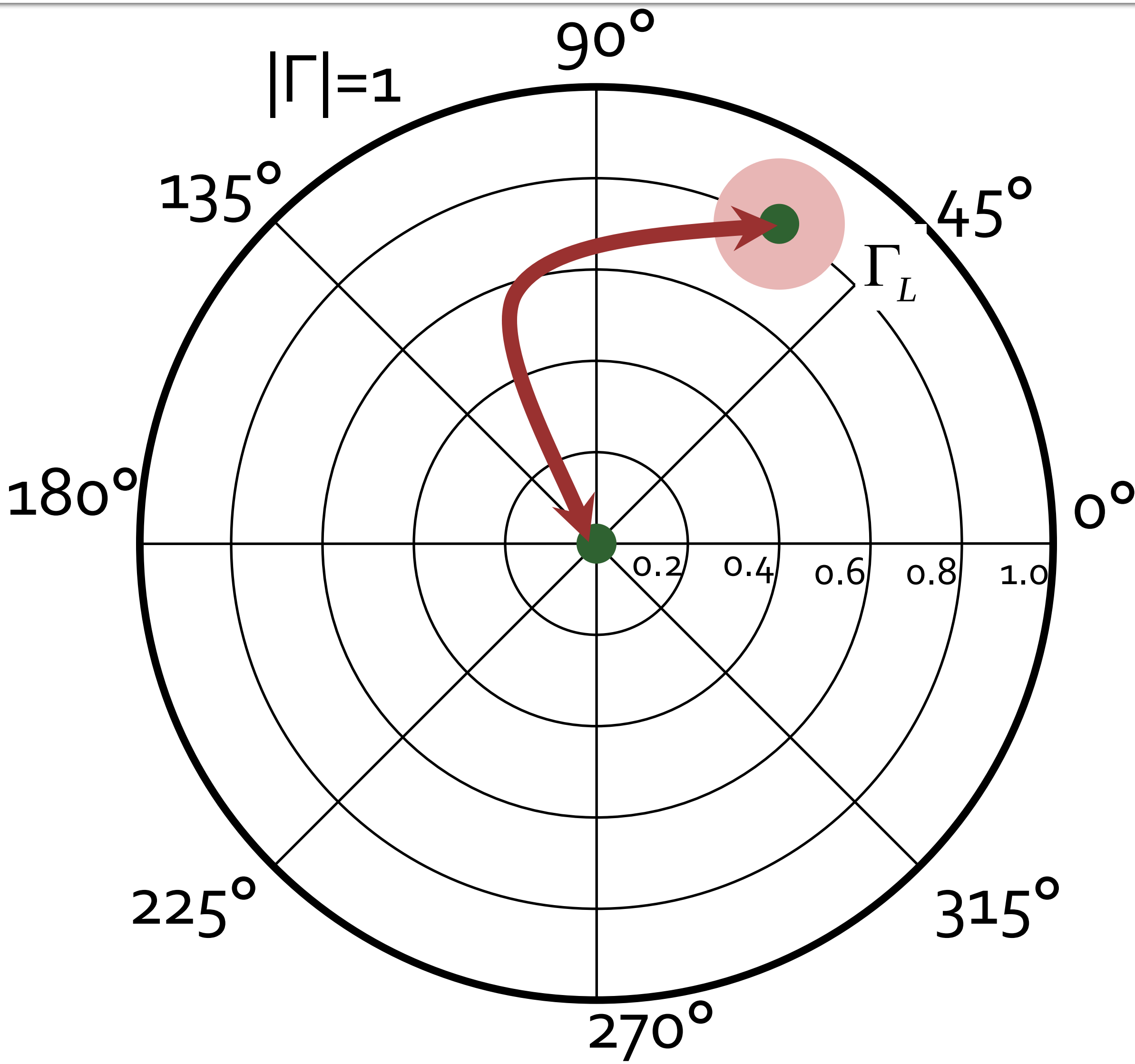
The source (eg. the transistor) having Z_X needs to see a certain reflection coefficient Γ_L towards the load Z_o

The matching circuit must move the point denoting the reflection coefficient in the area where for a Z_o load (Γ_o=0) we see towards it:

$\Gamma = \Gamma_L$ perfect match

$|\Gamma - \Gamma_L| \leq \Gamma_m$ "good enough" match

The Smith Chart, matching , $Z_L \neq Z_o$, $Z_L = Z_o$

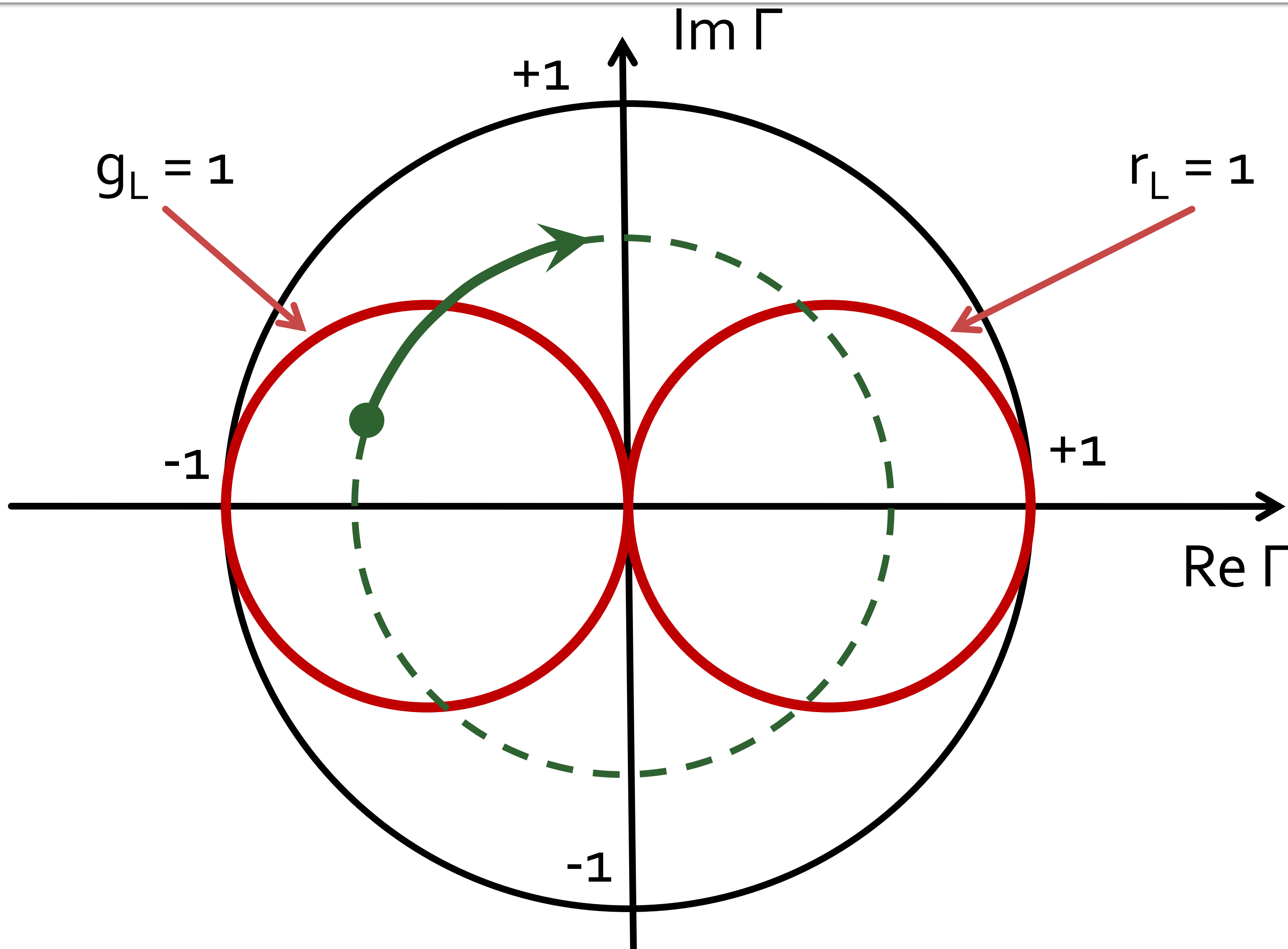


- The matching sections needed to move
  - Γ_L in Γ_o
  - Γ_o in Γ_L
- are **identical**. They differ only by the **order** in which the elements are introduced into the matching circuit
- As a result, we can use in match design the same:
  - **methods**
  - **formulae**

Impedance Matching

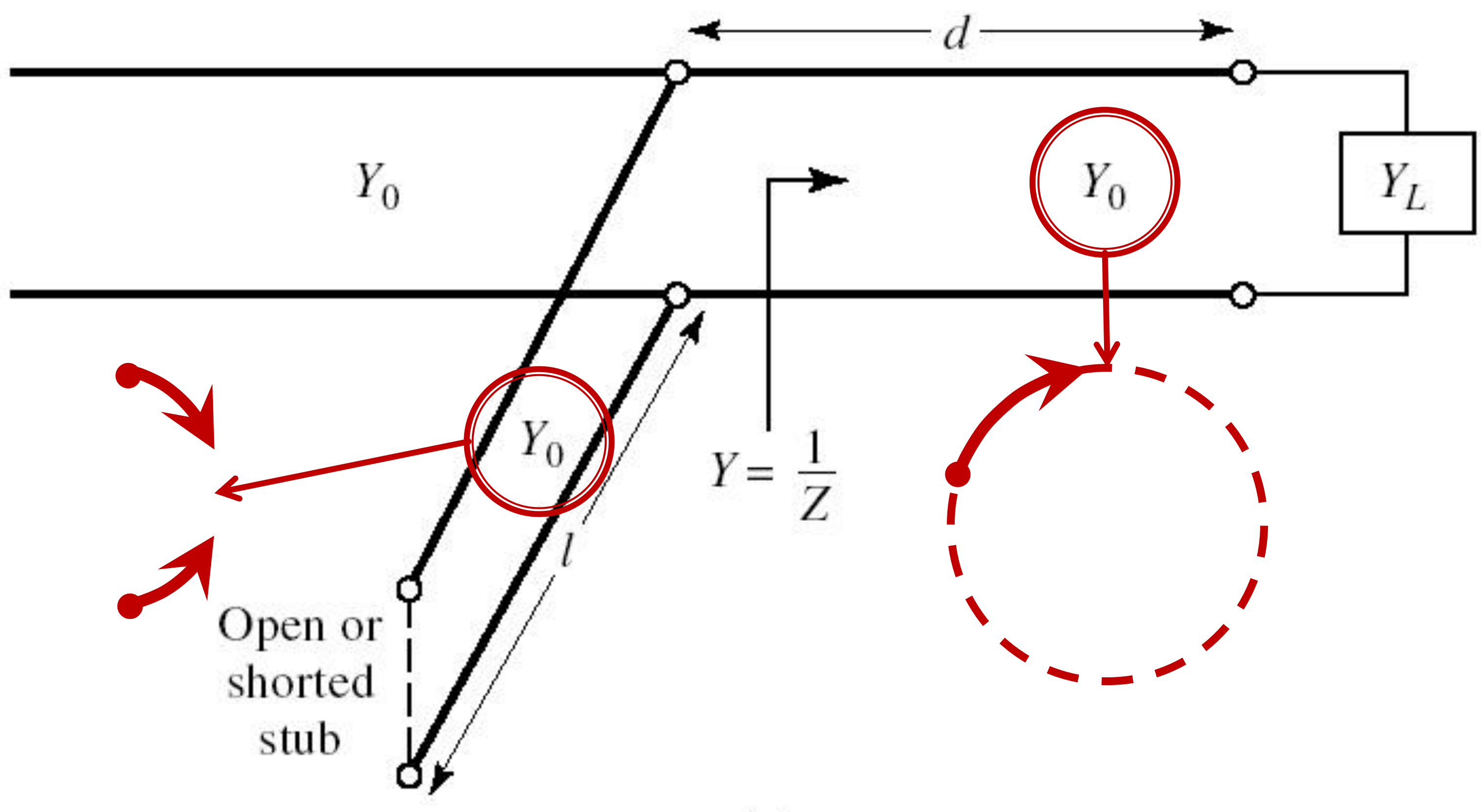
Impedance Matching with Stubs

Smith chart, r=1 and g=1



Case 1, Shunt Stub

- Shunt Stub



Analytical solution, usage

$$\boxed{\cos(\varphi + 2\theta) = -|\Gamma_S|}$$

$$\boxed{\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}}$$

$$\Gamma_S = 0.593\angle 46.85°$$

$$|\Gamma_S| = 0.593; \quad \varphi = 46.85° \qquad \cos(\varphi + 2\theta) = -0.593 \Rightarrow (\varphi + 2\theta) = \pm 126.35°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**

    $$(46.85° + 2\theta) = +126.35° \qquad \theta = +39.7° \qquad \operatorname{Im} y_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.472$$

    $$\theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = -55.8°(+180°) \rightarrow \theta_{sp} = 124.2°$$
  - **"-" solution**

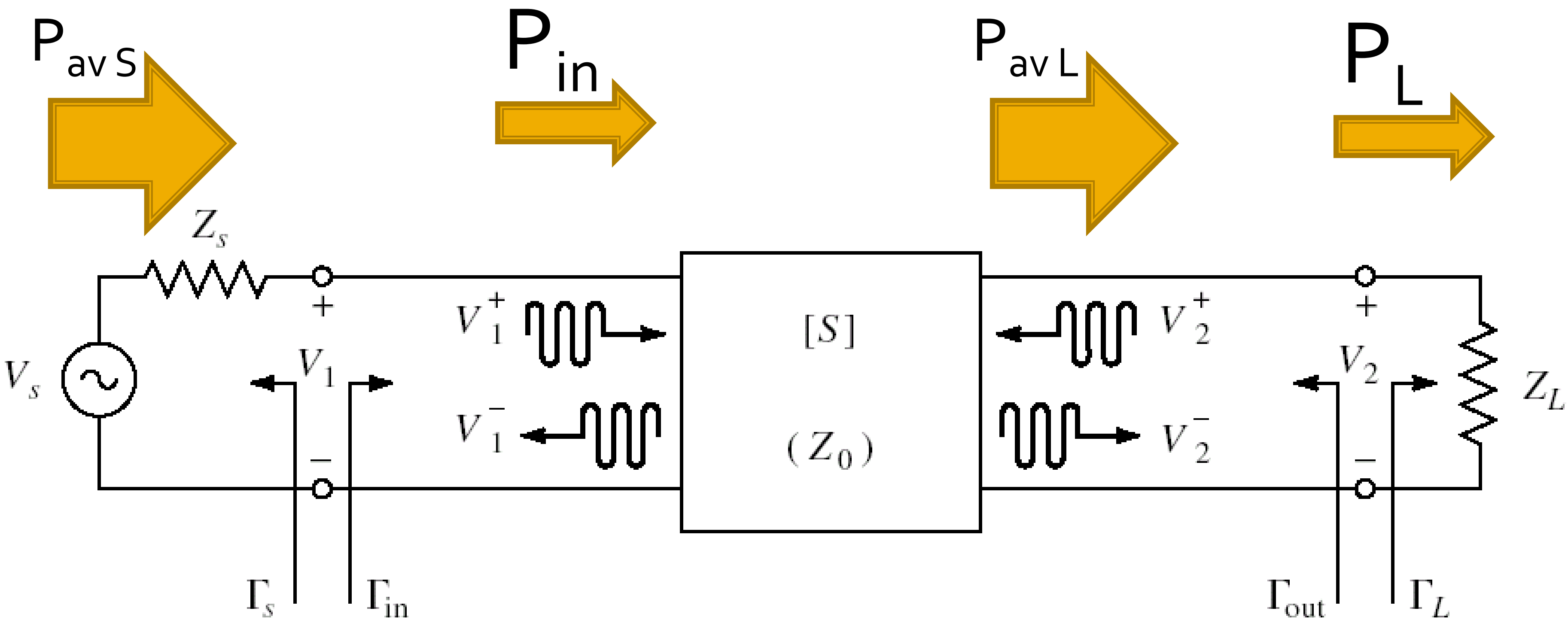
    $$(46.85° + 2\theta) = -126.35° \qquad \theta = -86.6°(+180°) \rightarrow \theta = 93.4°$$

    $$\operatorname{Im} y_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.472 \qquad \theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = 55.8°$$

Microwave Amplifiers

Power / Matching

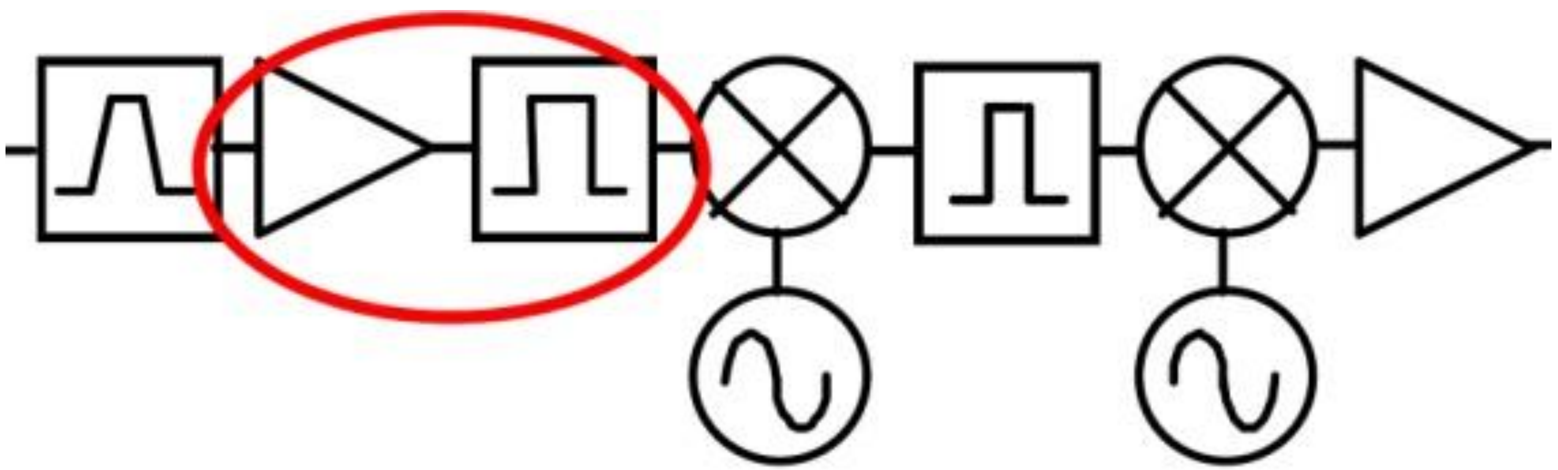
- Two ports in which matching influences the power transfer



Microwave Filters

Assignment

- this structure is frequently encountered in radiocommunication systems



Insertion loss method

$$P_{LR} = \frac{P_S}{P_L} = \frac{1}{1 - |\Gamma(\omega)|^2}$$

- $|\Gamma(\omega)|^2$ is an even function of ω

$$|\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2) + N(\omega^2)}$$

$$P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$

- Choosing M and N polynomials appropriately leads to a filter with a completely specified frequency response

Insertion loss method

- We control the power loss ratio/attenuation introduced by the filter:
  - in the passband (pass all frequencies)
  - in the stopband (reject all frequencies)

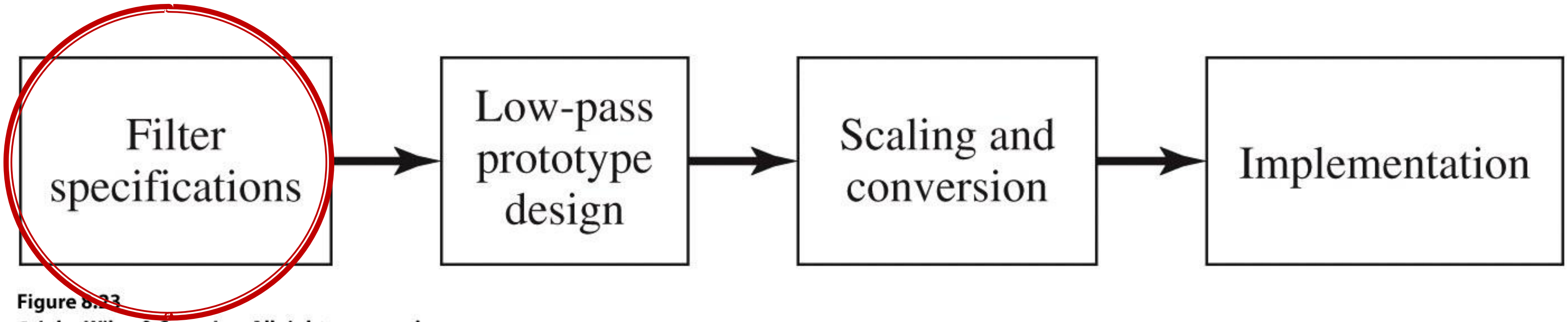
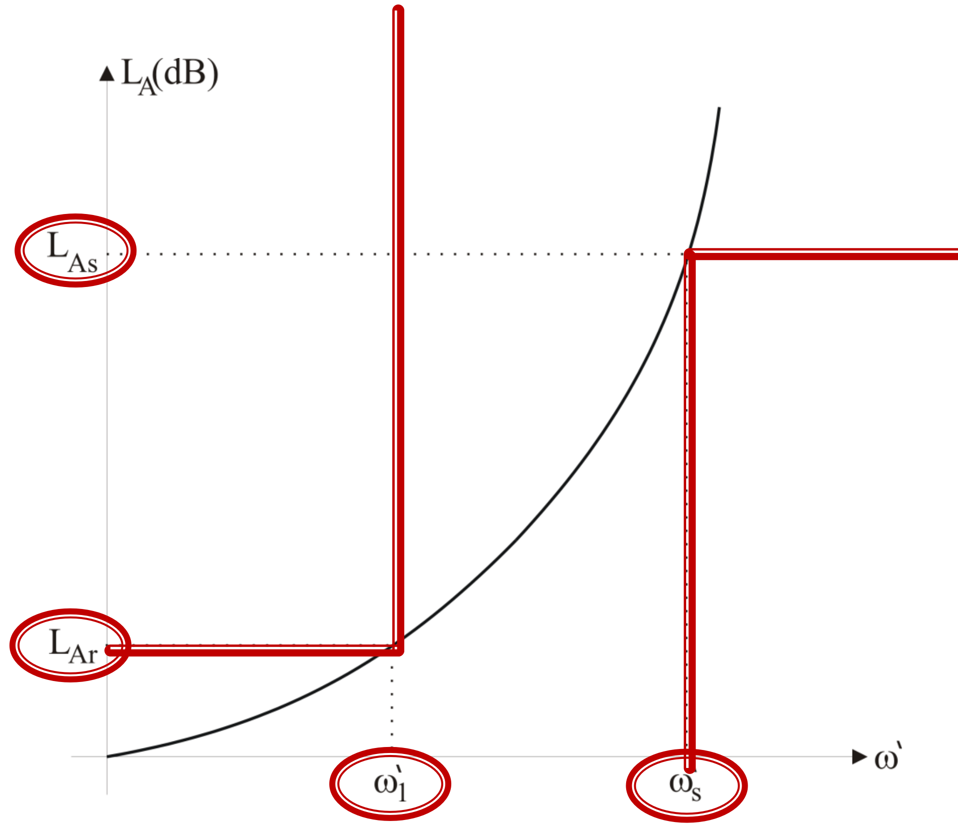


Figure 8.23

Filter specifications

- Attenuation
  - in passband
  - in stopband
  - most often in **dB**
- Frequency range
  - passband
  - stopband
  - cutoff frequency ω_1' usually normalized (**= 1**)



Insertion loss method

- We choose the right polynomials to design an **low-pass** filter (prototype)
- The low-pass prototype are then converted to the desired other types of filters
  - low-pass, high-pass, bandpass, or bandstop

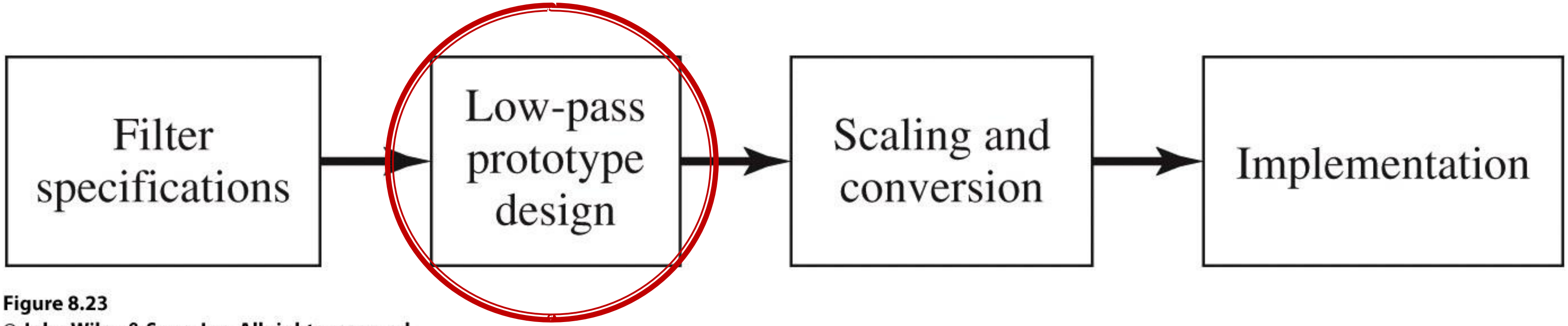


Figure 8.23

Maximally Flat/Equal ripple LPF Prototype

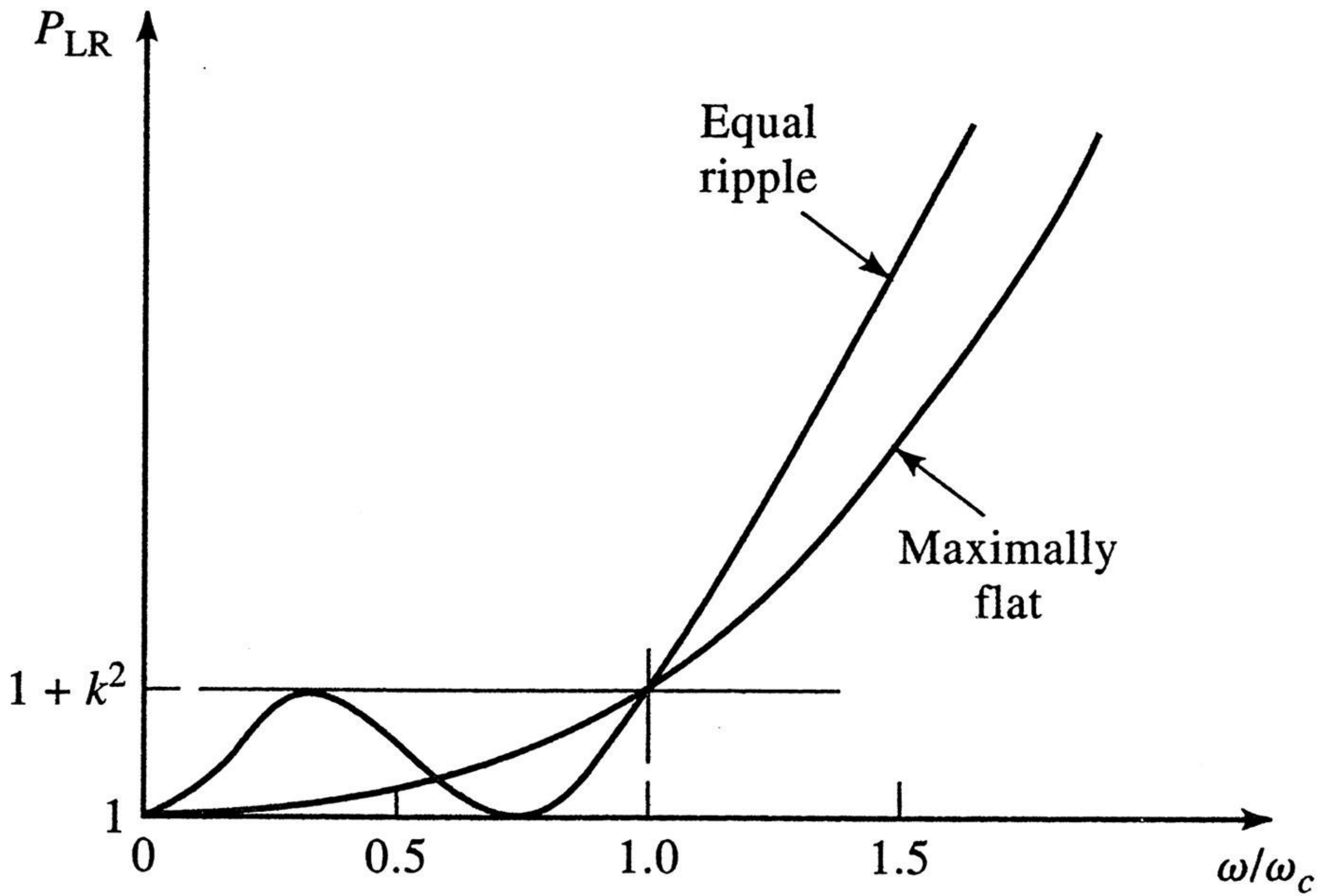
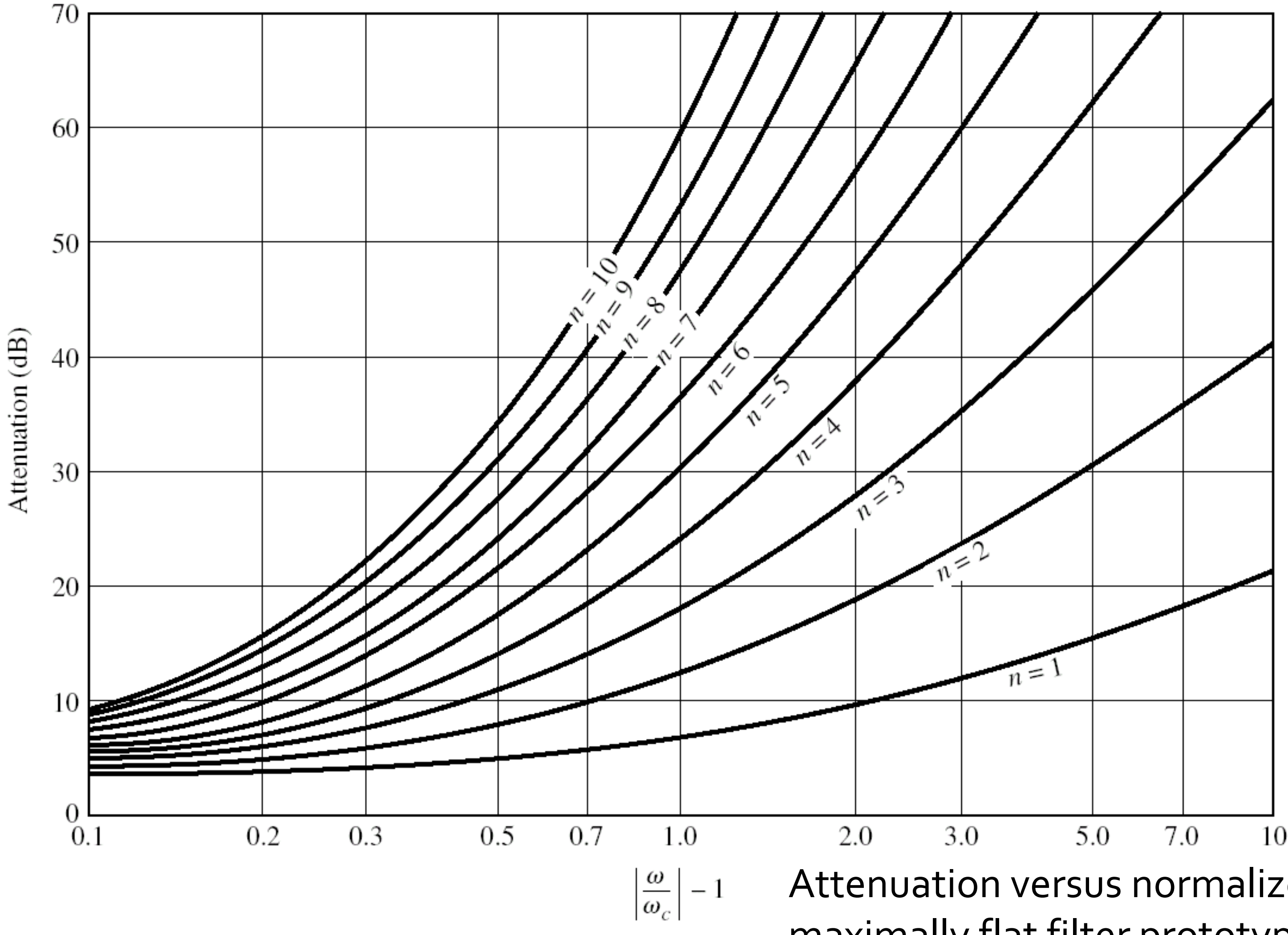


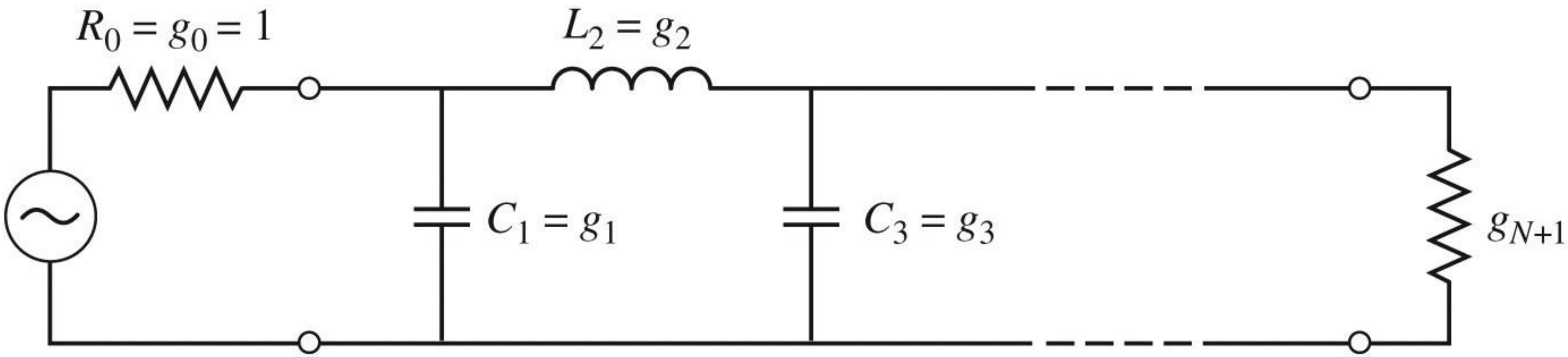
Figure 8.21

Maximally flat filter prototypes

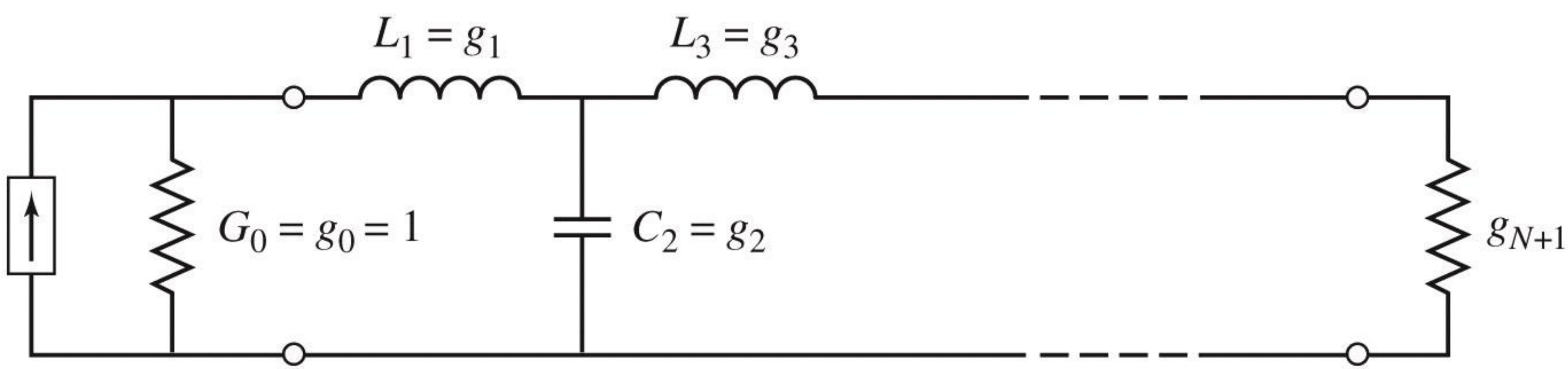


Attenuation versus normalized frequency for maximally flat filter prototypes

Prototype Filters



(a)



(b)

Prototype Filters

- Prototype filters are:
  - Low-Pass Filters (LPF)
  - cutoff frequency ω_o = 1 rad/s (f_o = 0.159 Hz)
  - connected to a source with R = 1Ω
- The number of reactive elements (L/C) is the order of the filter (N)
- Reactive elements are alternated: series L / shunt C
- There two prototypes with the same response, a prototype beginning with a shunt C element, and a prototype beginning with a series L element

Maximally Flat LPF Prototype

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

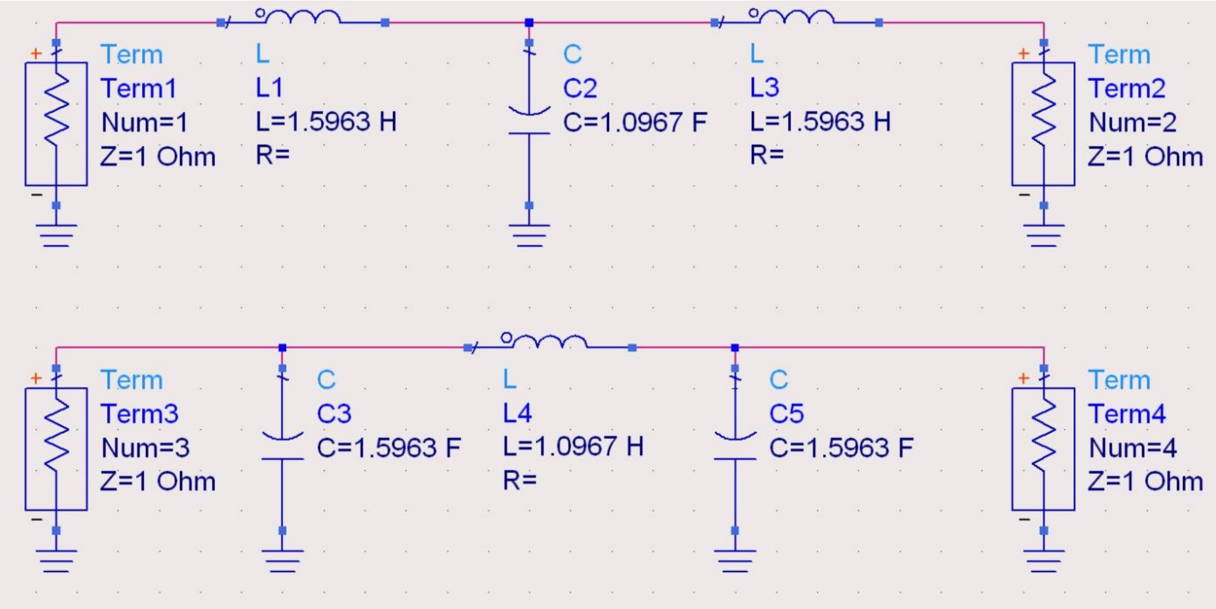
Table 8.3

Example

- Design a **3rd order** ~~bandpass~~ filter with **0.5 dB ripples** in passband. ~~The center frequency of the filter should be 1 GHz. The fractional bandwidth of the passband should be 10%, and the impedance 50Ω~~.

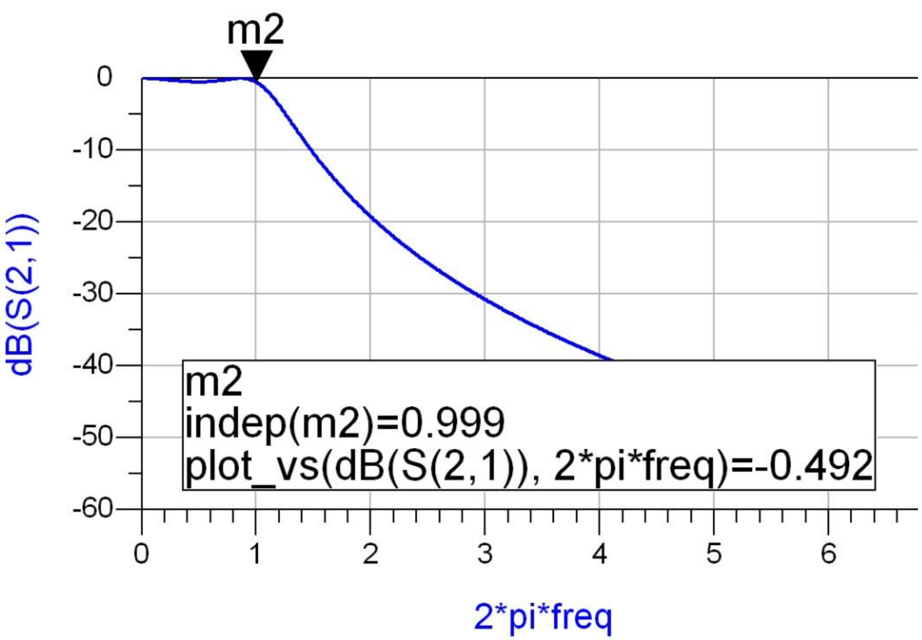
LPF Prototype

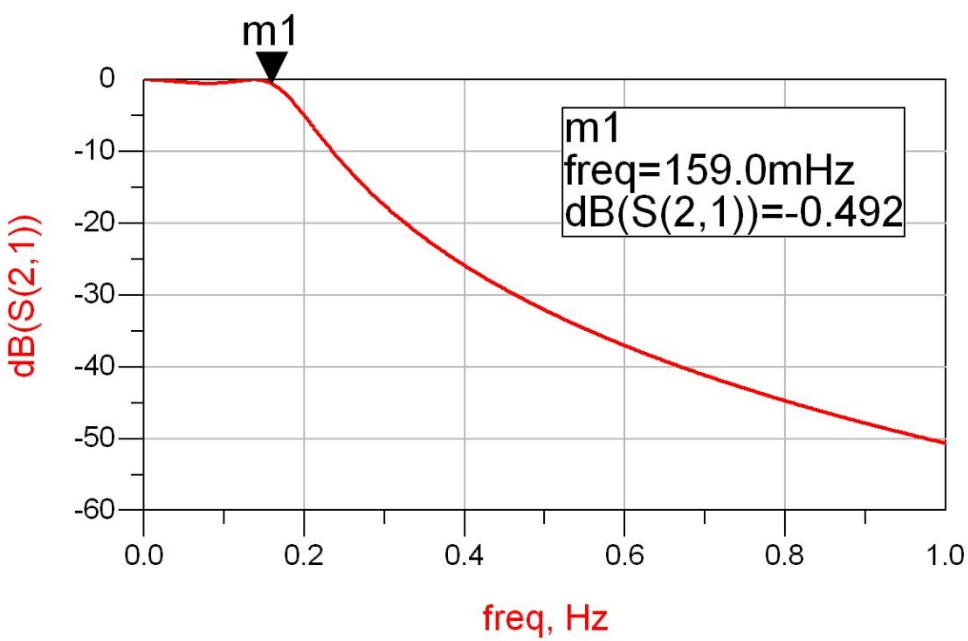
- 0.5dB equal-ripple table or design formulas:
  - g1 = 1.5963 = L1/C3,
  - g2 = 1.0967 = C2/L4,
  - g3 = 1.5963 = L3/C5,
  - g4=1.000 =R_L



LPF Prototype

- $\omega_o = 1$ rad/s ($f_o = \omega_o / 2\pi = 0.159$ Hz)





Continue

Impedance and Frequency Scaling

- After computing prototype filter's elements:
  - Low-Pass Filters (LPF)
  - cutoff frequency ω_o = 1 rad/s (f_o = 0.159 Hz)
  - connected to a source with R = 1Ω
- component values can be scaled in terms of impedance and frequency

Impedance and Frequency Scaling

- LPF Prototype is only used as an intermediate step
  - Low-Pass Filter (LPF)
  - cutoff frequency ω_o = 1 rad/s (f_o = 0.159 Hz)
  - connected to a source with R = 1Ω

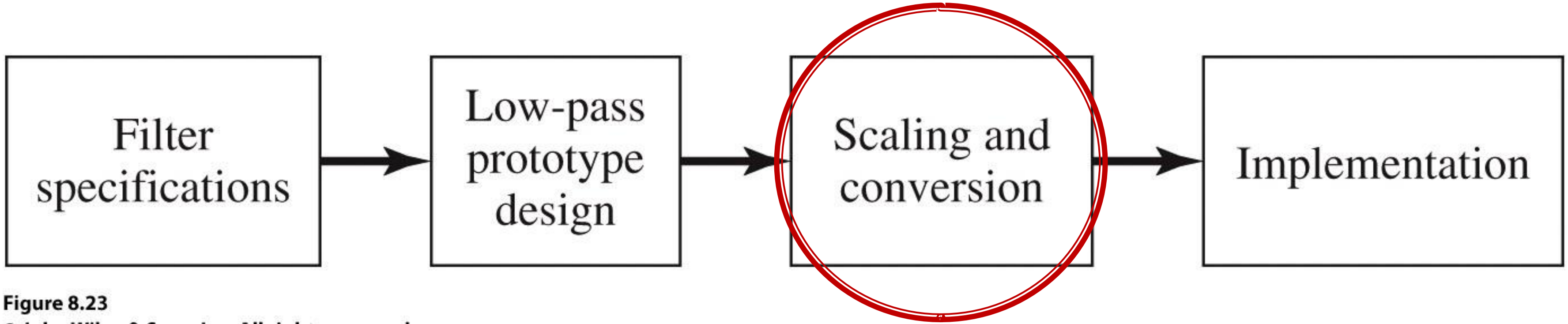


Figure 8.23

Impedance Scaling

- To design a filter which will work with a source resistance of R_o we multiplying all the impedances of the prototype design by R_o (" ' " denotes scaled values)

$$R_s' = R_0 \cdot (R_s = 1)$$

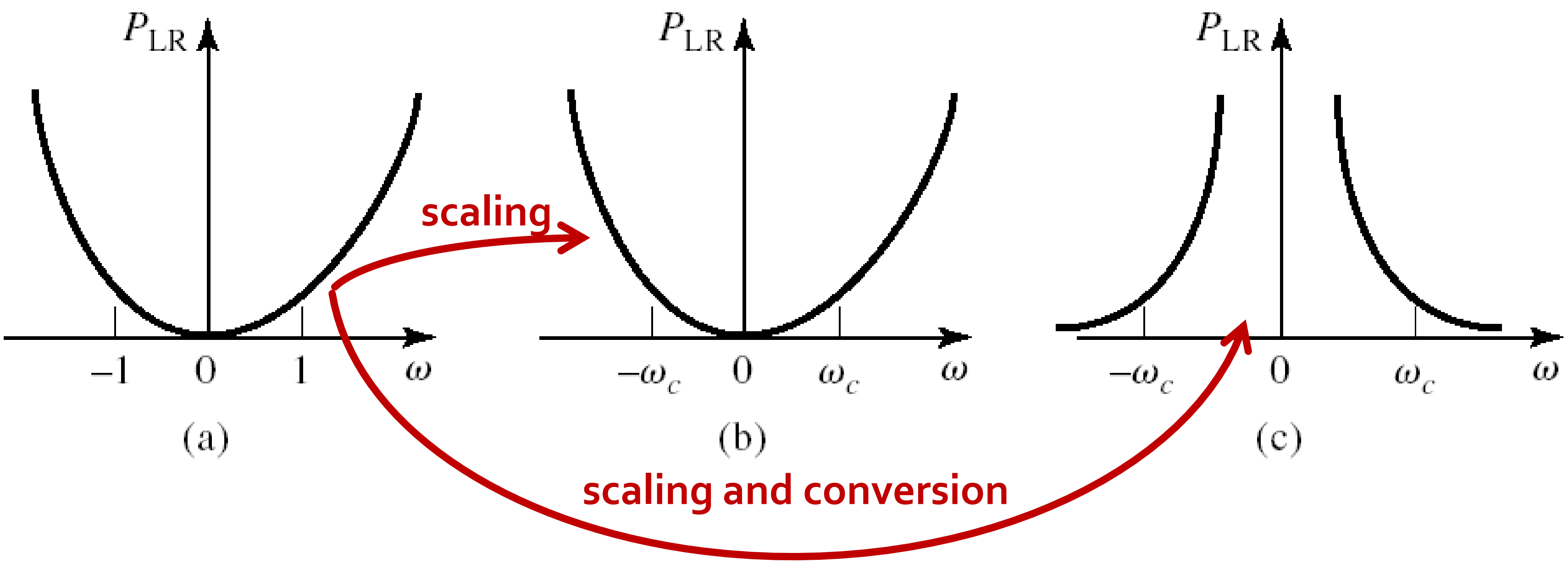
$$R_L' = R_0 \cdot R_L$$

$$L' = R_0 \cdot L$$

$$C' = \frac{C}{R_0}$$

Frequency Scaling

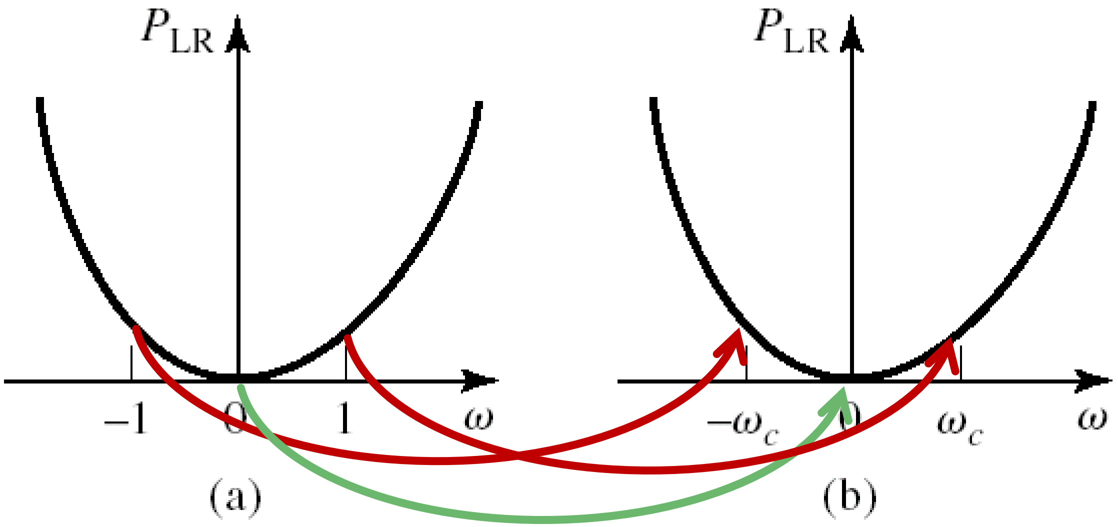
- changing the cutoff frequency – (fig. b)
- changing the type (for example LPF → HPF – fig. c) requires also conversion



Frequency Scaling

- To change the cutoff frequency of a low-pass prototype from unity to ω_c we apply a variable substitution

$$\omega \leftarrow \frac{\omega}{\omega_c}$$



Frequency Scaling

- To change the cutoff frequency of a low-pass prototype we apply a variable substitution:

$$\omega \leftarrow \frac{\omega}{\omega_c}$$

- Equivalent to the widening of the power loss filter response

$$P'_{LR}(\omega) = P_{LR}\left(\frac{\omega}{\omega_c}\right)$$

$$j \cdot X_k = j \cdot \frac{\omega}{\omega_c} \cdot L_k = j \cdot \omega \cdot L'_k \qquad j \cdot B_k = j \cdot \frac{\omega}{\omega_c} \cdot C_k = j \cdot \omega \cdot C'_k$$

Frequency Scaling LPF → LPF

- New element values for frequency scaling:

$$L_k' = \frac{L_k}{\omega_c} \qquad C_k' = \frac{C_k}{\omega_c}$$

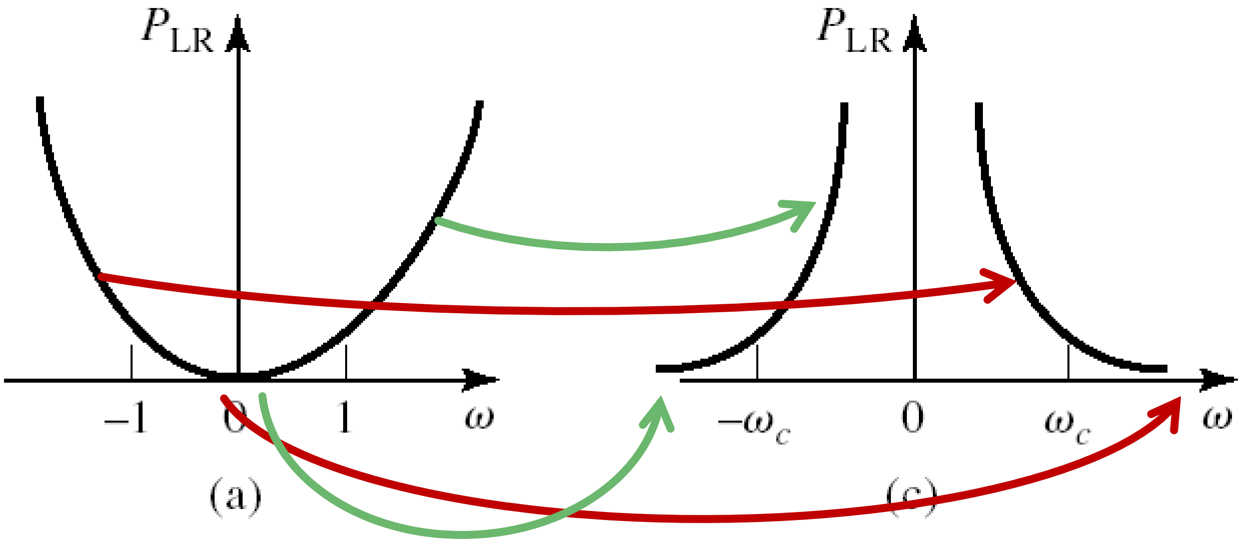
- When both impedance and frequency scaling are required:

$$L_k' = \frac{R_0 \cdot L_k}{\omega_c} \qquad C_k' = \frac{C_k}{R_0 \cdot \omega_c}$$

Low-pass to high-pass transformation LPF → HPF

- Variable substitution for LPF → HPF:

$$\omega \leftarrow -\frac{\omega_c}{\omega}$$



High-pass transformation LPF → HPF

- Variable substitution for LPF → HPF :

$$\omega \leftarrow -\frac{\omega_c}{\omega}$$

$$j \cdot X_k = -j \cdot \frac{\omega_c}{\omega} \cdot L_k = \frac{1}{j \cdot \omega \cdot C_k'} \qquad j \cdot B_k = -j \cdot \frac{\omega_c}{\omega} \cdot C_k = \frac{1}{j \cdot \omega \cdot L_k'}$$

- Impedance scaling can be included

$$C_k' = \frac{1}{R_0 \cdot \omega_c \cdot L_k} \qquad L_k' = \frac{R_0}{\omega_c \cdot C_k}$$

- In the schematic series **inductors** must be replaced with series **capacitors**, and shunt **capacitors** must be replaced with shunt **inductors**

Bandpass Transformation LPF → BPF

- Variable substitution for LPF → BPF:

$$\omega \leftarrow \frac{\omega_0}{\omega_2 - \omega_1}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

- where we use the fractional bandwidth of the passband and the center frequency

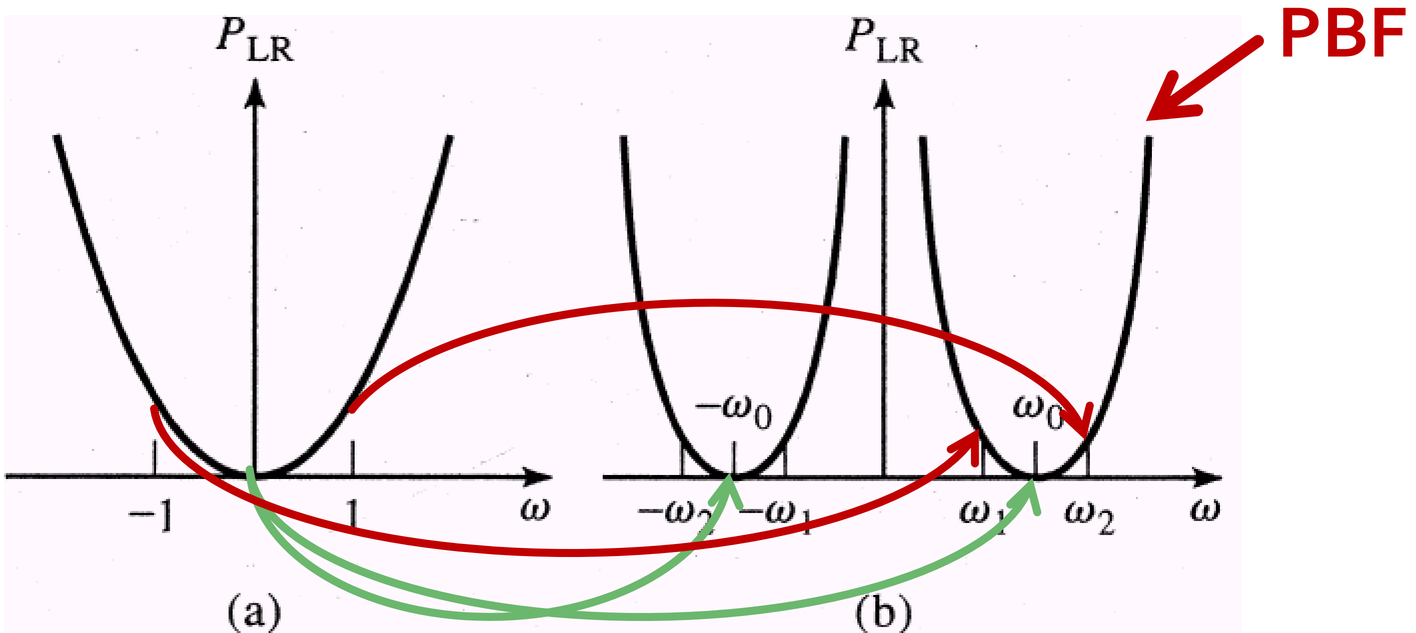
$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0} \qquad \omega_0 = \sqrt{\omega_1 \cdot \omega_2}$$

Bandpass Transformation LPF → BPF

$$\omega=\omega_0 \rightarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=\frac{1}{\Delta}\left(\frac{\omega_0}{\omega_0}-\frac{\omega_0}{\omega_0}\right)=0 \qquad \omega=-\omega_0 \rightarrow \frac{1}{\Delta}\left(\frac{-\omega_0}{\omega_0}-\frac{\omega_0}{-\omega_0}\right)=0$$

$$\omega=\omega_1 \rightarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=\frac{1}{\Delta}\left(\frac{\omega_1^2-\omega_0^2}{\omega_0\cdot\omega_1}\right)=-1$$

$$\omega=\omega_2 \rightarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=\frac{1}{\Delta}\left(\frac{\omega_2^2-\omega_0^2}{\omega_0\cdot\omega_2}\right)=1$$



Bandpass Transformation LPF → BPF

$$j \cdot X_k = \frac{j}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \cdot L_k = j \cdot \frac{\omega \cdot L_k}{\Delta \cdot \omega_0} - j \cdot \frac{\omega_0 \cdot L_k}{\Delta \cdot \omega} = j \cdot \omega \cdot L_k' - j \frac{1}{\omega \cdot C_k'}$$

$$j \cdot B_k = \frac{j}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \cdot C_k = j \cdot \frac{\omega \cdot C_k}{\Delta \cdot \omega_0} - j \cdot \frac{\omega_0 \cdot C_k}{\Delta \cdot \omega} = j \cdot \omega \cdot C_k' - j \frac{1}{\omega \cdot L_k'}$$

- A series **inductor** in the prototype filter is transformed to a **series LC** circuit **in series**

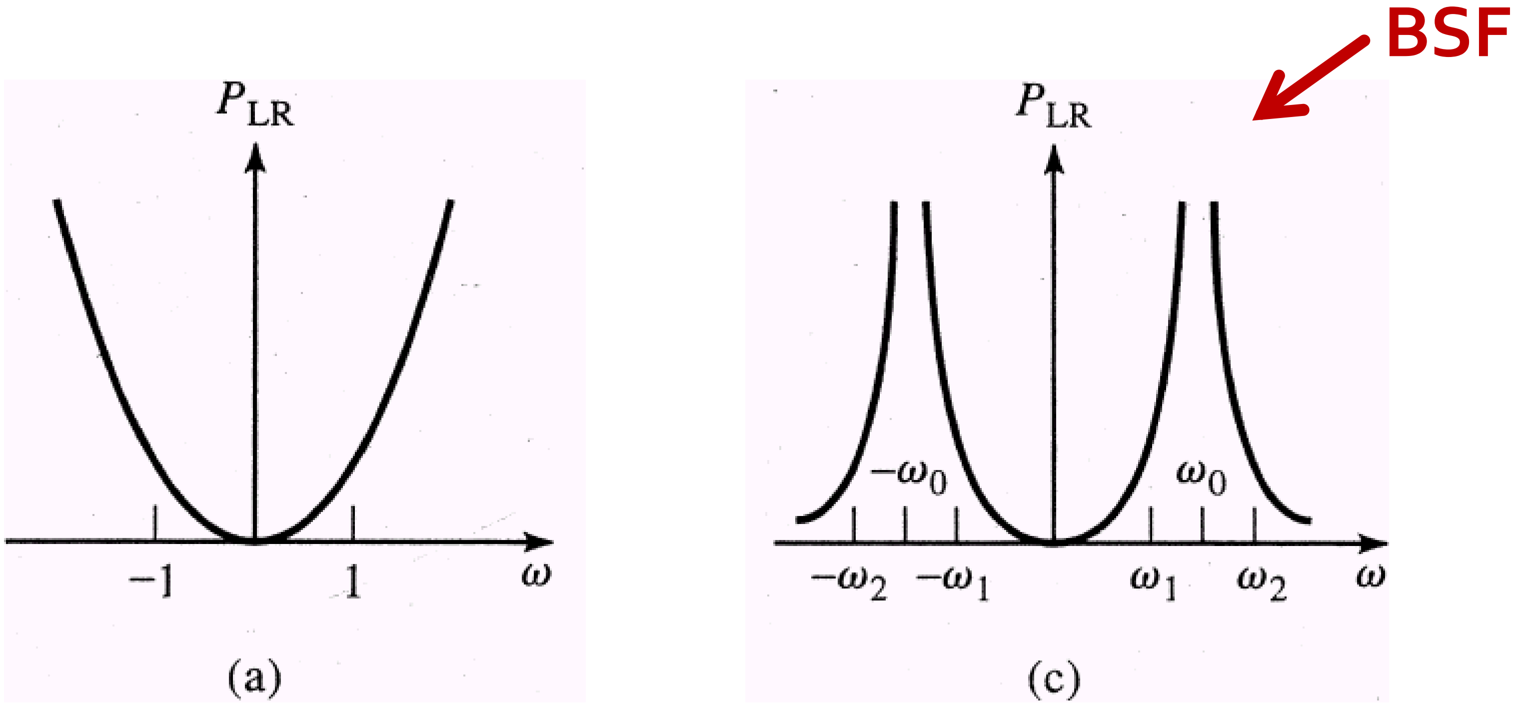
$$L_k' = \frac{L_k}{\Delta \cdot \omega_0} \qquad C_k' = \frac{\Delta}{\omega_0 \cdot L_k}$$

- A shunt **capacitor** in the prototype filter is transformed to a **shunt LC** circuit **in parallel**

$$L_k' = \frac{\Delta}{C_k \cdot \omega_0} \qquad C_k' = \frac{C_k}{\omega_0 \cdot \Delta}$$

Bandstop Transformation LPF → BSF

$$\omega \leftarrow -\Delta \cdot \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^{-1} \qquad \omega = \omega_0 \rightarrow \frac{-\Delta}{\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)} = \frac{-\Delta}{\left(\frac{\omega_0}{\omega_0} - \frac{\omega_0}{\omega_0} \right)} \rightarrow \pm\infty$$



Bandstop Transformation LPF → BSF

$$\omega \leftarrow -\Delta \cdot \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^{-1}$$

- A series **inductor** in the prototype filter is transformed to a **shunt LC** circuit **in series**

$$L'_k = \frac{\Delta \cdot L_k}{\omega_0} \qquad C'_k = \frac{1}{\omega_0 \cdot \Delta \cdot L_k}$$

- A shunt **capacitor** in the prototype filter is transformed to a **series LC** circuit **in parallel**

$$L'_k = \frac{1}{\Delta \cdot \omega_0 \cdot C_k} \qquad C'_k = \frac{\Delta \cdot C_k}{\omega_0}$$

Summary of Prototype Filter Transformations

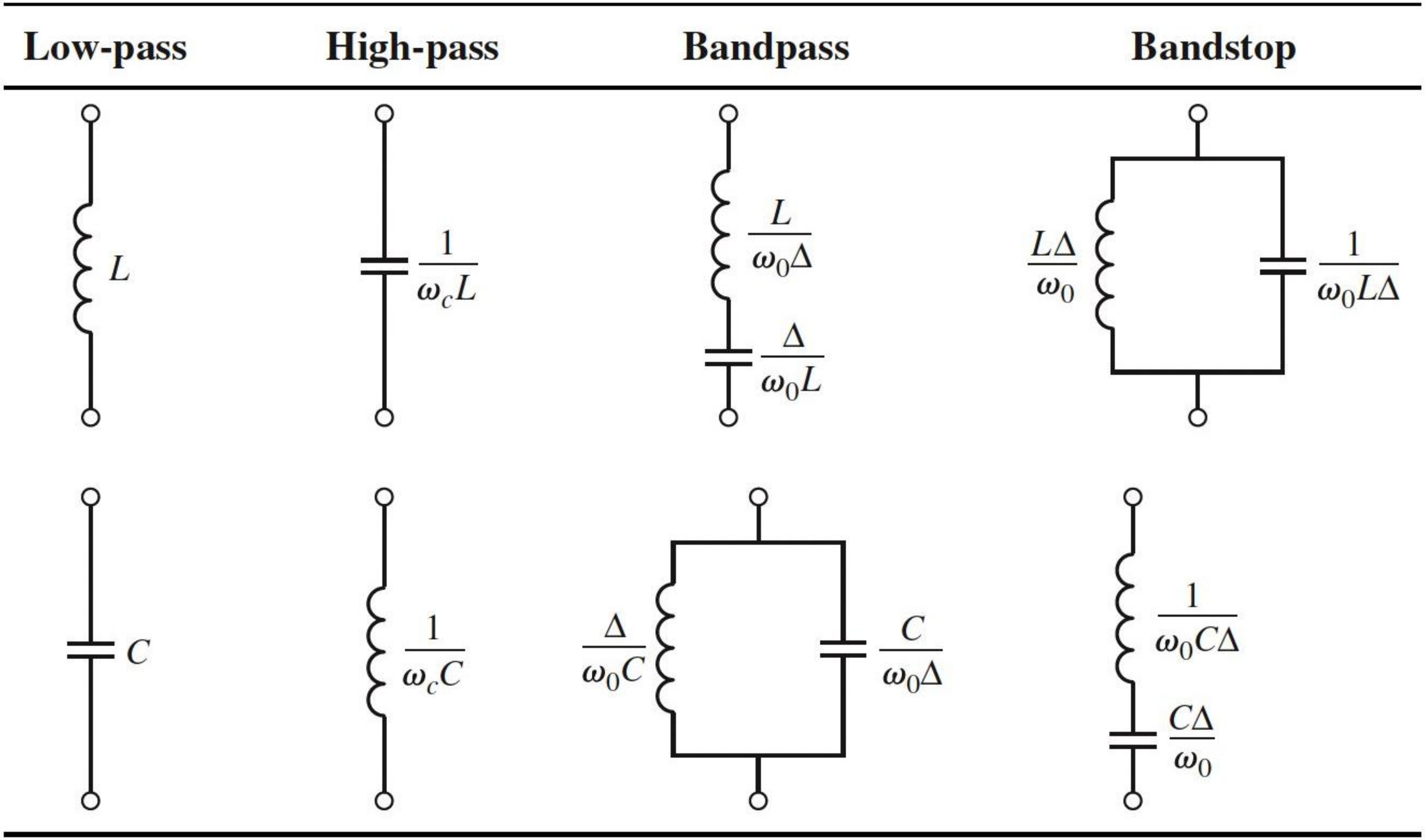


Table 8.6

Example

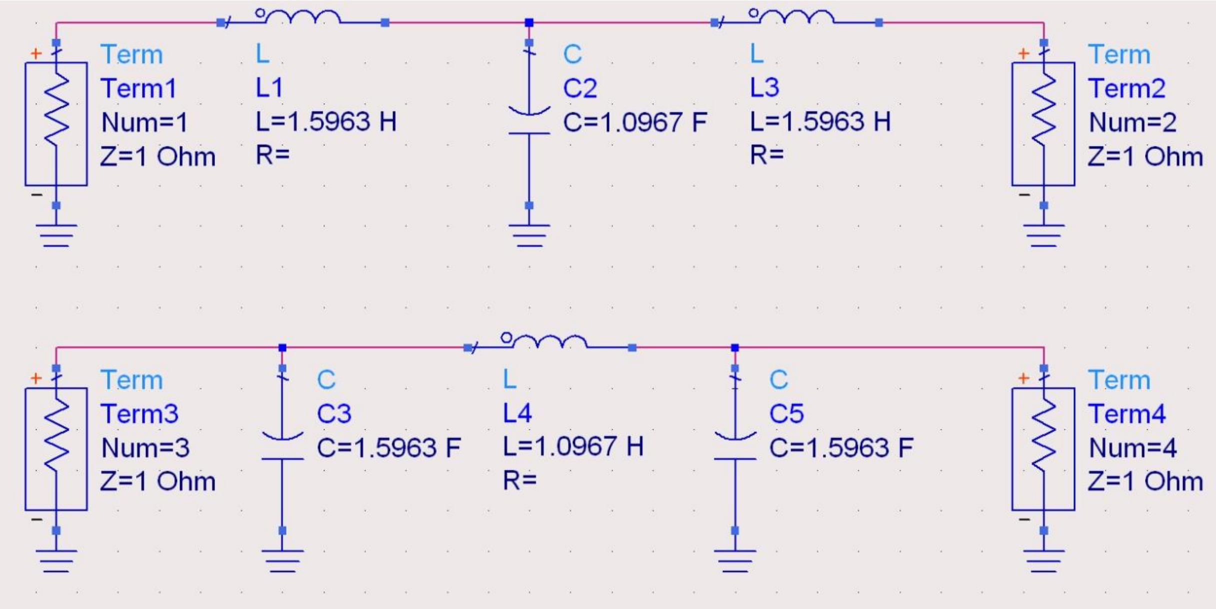
- Design a 3rd order **bandpass** filter with 0.5 dB ripples in passband. The **center frequency** of the filter should be 1 GHz. The **fractional bandwidth** of the passband should be 10%, and the **impedance** 50Ω.

$$\omega_0 = 2 \cdot \pi \cdot 1GHz = 6.283 \cdot 10^9 \, rad / s$$

$$\Delta = 0.1$$

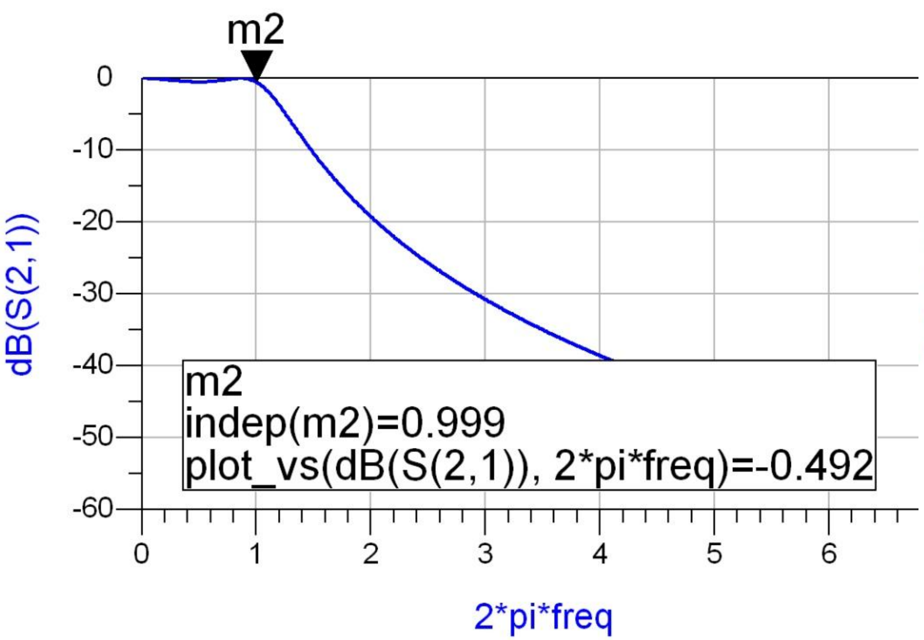
LPF Prototype

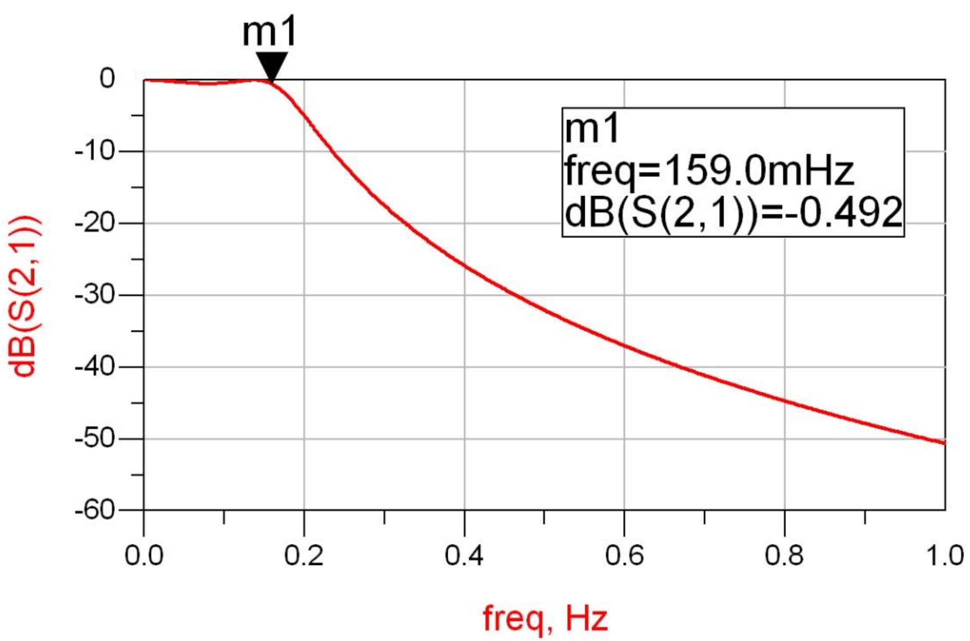
- 0.5dB equal-ripple table or design formulas:
  - g1 = 1.5963 = L1/C3,
  - g2 = 1.0967 = C2/L4,
  - g3 = 1.5963 = L3/C5,
  - g4=1.000 =R_L



LPF Prototype

- ω_o = 1 rad/s ($f_o = \omega_o / 2\pi$ = 0.159 Hz)





Bandpass Transformation / BPF

$$\omega_0 = 2 \cdot \pi \cdot 1GHz = 6.283 \cdot 10^9 \, rad/s \qquad \Delta = \frac{\Delta\omega}{\omega_0} = \frac{\Delta f}{f_0} = 0.1 \qquad R_0 = 50\,\Omega$$

g1 = 1.5963 = L1, g3 = 1.5963 = L3,

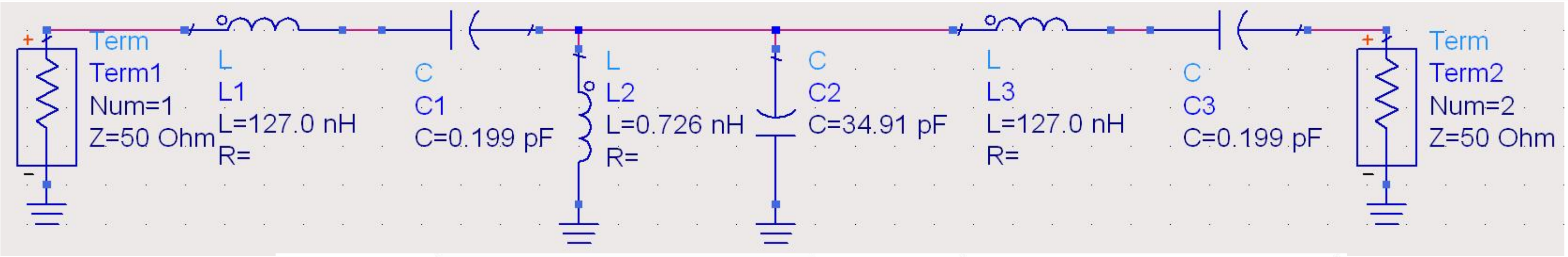
g2 = 1.0967 = C2, g4=1.000 =R_L

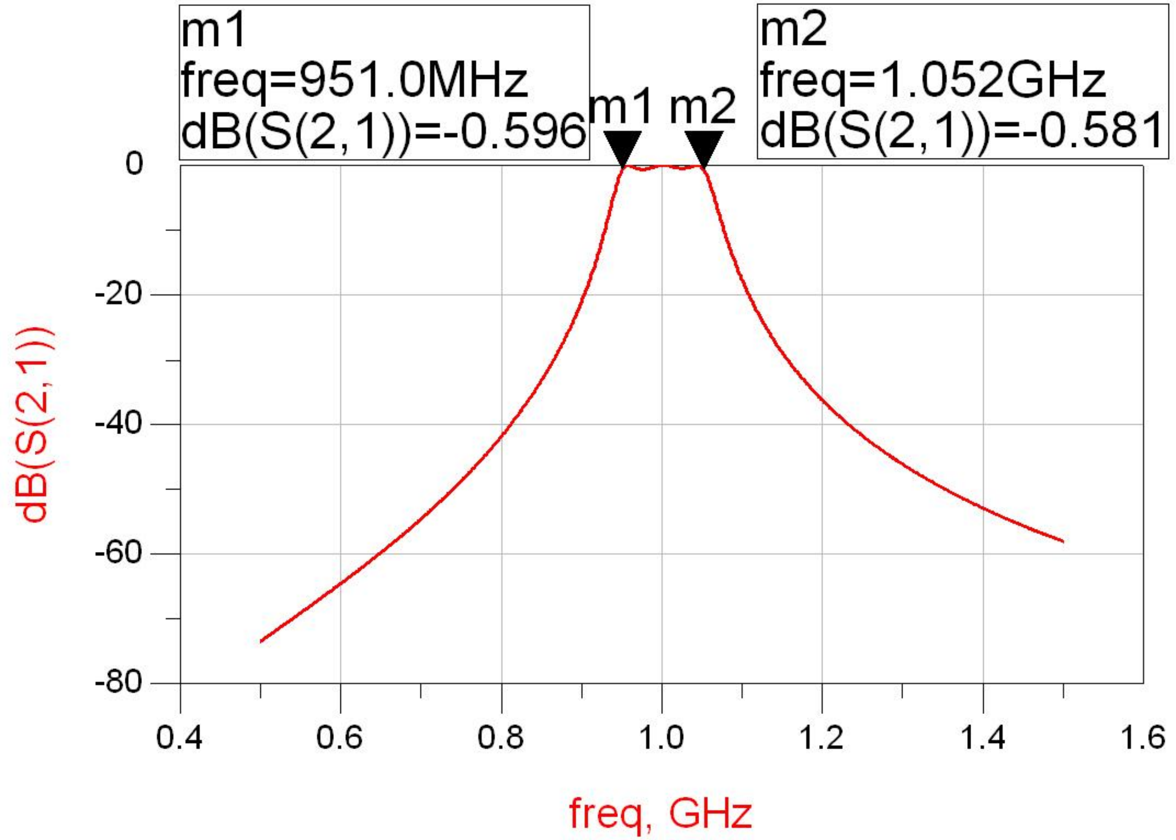
$$L_1' = \frac{L_1 \cdot R_0}{\Delta \cdot \omega_0} = 127.0\,nH \qquad C_1' = \frac{\Delta}{\omega_0 \cdot L_1 \cdot R_0} = 0.199\,pF$$

$$L_2' = \frac{\Delta \cdot R_0}{\omega_0 \cdot C_2} = 0.726\,nH \qquad C_2' = \frac{C_2}{\Delta \cdot \omega_0 \cdot R_0} = 34.91\,pF$$

$$L_3' = \frac{L_3 \cdot R_0}{\Delta \cdot \omega_0} = 127.0\,nH \qquad C_3' = \frac{\Delta}{\omega_0 \cdot L_3 \cdot R_0} = 0.199\,pF$$

ADS


Term
Term1
Num=1
Z=50 Ohm
L
L1
L=127.0 nH
R=
C
C1
C=0.199 pF
L
L2
L=0.726 nH
R=
C
C2
C=34.91 pF
L
L3
L=127.0 nH
R=
C
C3
C=0.199 pF
Term
Term2
Num=2
Z=50 Ohm


m1
freq=951.0MHz
dB(S(2,1))=-0.596
m2
freq=1.052GHz
dB(S(2,1))=-0.581
m1
m2
0
-20
-40
-60
-80
dB(S(2,1))
0.4
0.6
0.8
1.0
1.2
1.4
1.6
freq, GHz

Microwave Filters Implementation

Microwave Filters Implementation

- The lumped-element (L, C) filter design generally works well **only** at low frequencies (RF):
  - lumped-element inductors and capacitors are generally available only for a limited range of values, and can be difficult to implement at microwave frequencies
  - difficulty to obtain the (very low) required tolerance for elements

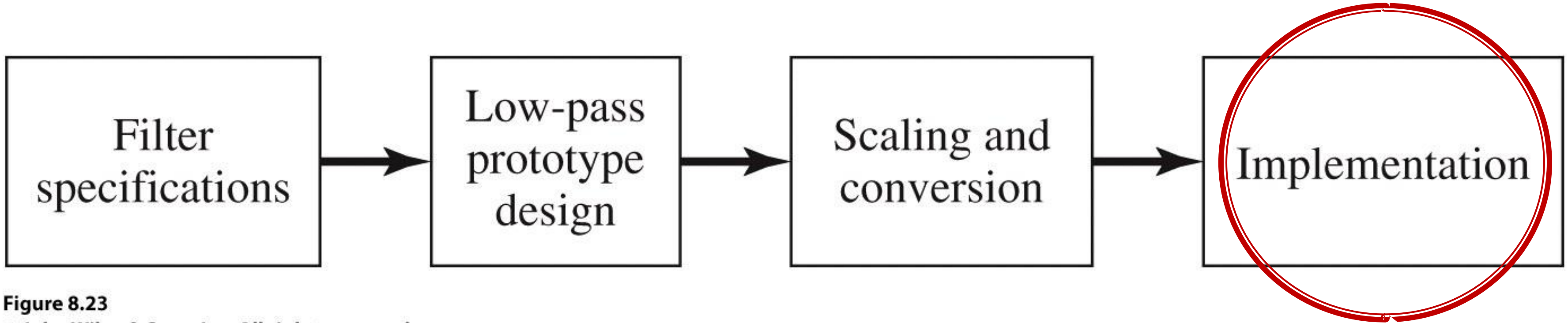


Figure 8.23

Richards' Transformation

- Impedance seen at the input of a line loaded with Z_L

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

- We prefer the load impedance to be:
  - open circuit ($Z_L = \infty$) $\quad Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l$
  - short circuit ($Z_L = 0$) $\quad Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l$
- Input impedance is:
  - capacitive $\quad Z_{in,oc} = j \cdot X_C = \frac{1}{j \cdot B_C} \qquad Z_0 \leftrightarrow \frac{1}{C} \quad \tan \beta \cdot l \leftrightarrow \omega$
  - inductive $\quad Z_{in,sc} = j \cdot X_L \qquad Z_0 \leftrightarrow L \quad \tan \beta \cdot l \leftrightarrow \omega$

Richards' Transformation

- Variable substitution

$$\Omega = \tan \beta \cdot l = \tan\left(\frac{\omega \cdot l}{v_p}\right)$$

- With this variable substitution we define:
  - reactance of an inductor

$$j \cdot X_L = j \cdot \Omega \cdot L = j \cdot L \cdot \tan \beta \cdot l$$

  - susceptance of a capacitor

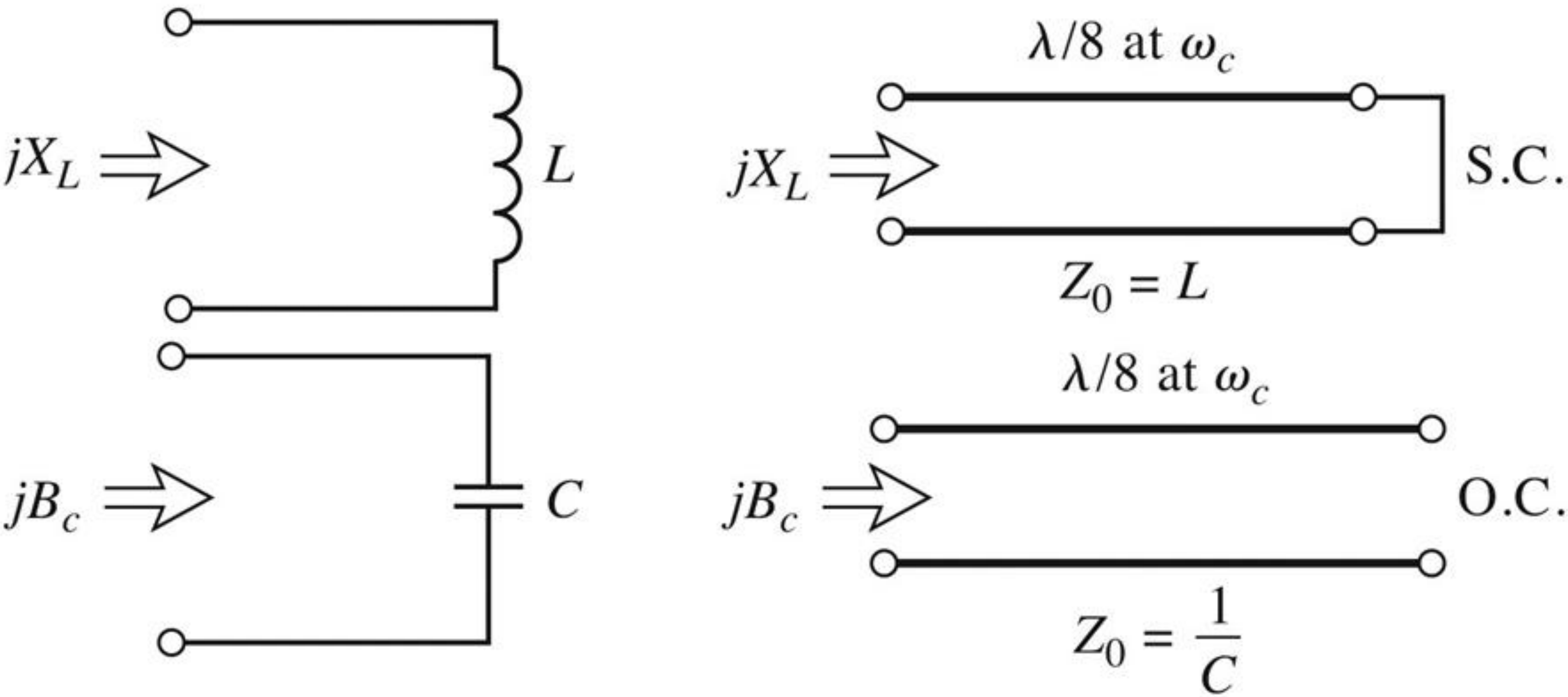
$$j \cdot B_C = j \cdot \Omega \cdot C = j \cdot C \cdot \tan \beta \cdot l$$

- The equivalent filter in Ω has a cutoff frequency at:

$$\Omega = 1 = \tan \beta \cdot l \quad \rightarrow \quad \beta \cdot l = \frac{\pi}{4} \quad \rightarrow \quad l = \frac{\lambda}{8}$$

Richards' Transformation

- allows implementation of the inductors and capacitors with lines **after** the transformation of the LPF prototype to the required type (LPF/HPF/BPF/BSF)



Richards' Transformation

- By choosing the open-circuited or short-circuited lines to be $\lambda/8$ at the desired cutoff frequency (ω_c) and the corresponding characteristic impedances (L/C from LPF prototype) we will obtain at frequencies around ω_c a behavior similar to that of the prototype filter.
  - At frequencies far from ω_c the behavior of the filter will no longer be identical to that of the prototype (in specific situations the correct behavior must be **verified**)
  - Frequency scaling is simplified: choosing the appropriate physical length of the line to have the electrical length $\lambda/8$ at the desired cutoff frequency
- All lines will have equal electrical lengths ($\lambda/8$) and thus comparable physical lengths, so the lines are called **commensurate** lines

Richards' Transformation

- At the frequency $\omega = 2 \cdot \omega_c$ the lines will be $\lambda/4$ long

$$l = \frac{\lambda}{4} \quad \Rightarrow \quad \beta \cdot l = \frac{\pi}{2} \quad \Rightarrow \quad \tan \beta \cdot l \to \infty$$

- an supplemental attenuation pole will occur at $2 \cdot \omega_c$ (LPF):
  - inductances (usually in series) $Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l \to \infty$
  - capacitances (usually shunt) $Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l \to 0$

Richards' Transformation

- the periodicity of tan function implies the periodicity of the filter implemented with lines
  - the filter response will be repeated every $4 \cdot \omega_c$

$$\tan(\alpha + \pi) = \tan\alpha$$

$$\beta \cdot l\big|_{\omega=\omega_c} = \frac{\pi}{4} \quad \Rightarrow \quad \frac{\omega_c \cdot l}{v_p} = \frac{\pi}{4} \quad \Rightarrow \quad \pi = \frac{(4 \cdot \omega_c) \cdot l}{v_p}$$

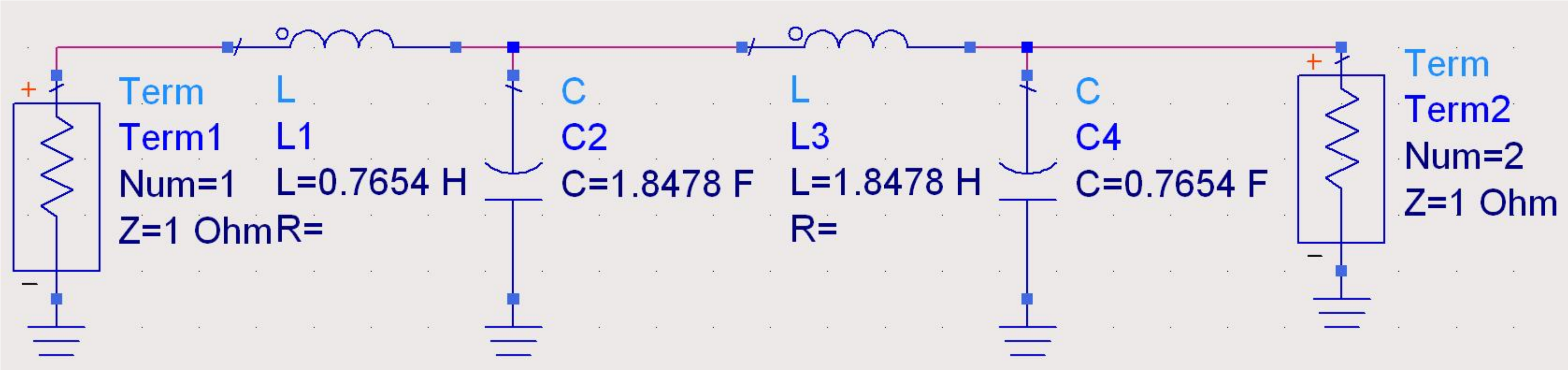
$$Z_{in}(\omega) = Z_{in}(\omega + 4 \cdot \omega_c) \quad \Rightarrow \quad P_{LR}(\omega) = P_{LR}(\omega + 4 \cdot \omega_c)$$

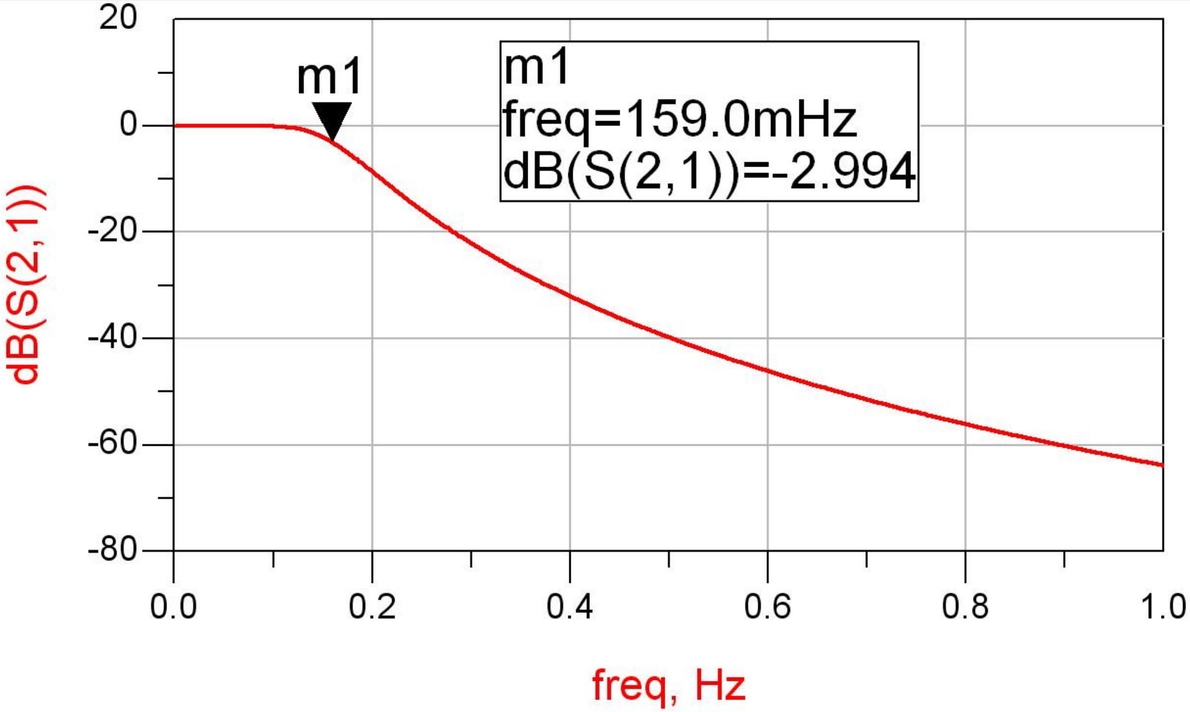
$$P_{LR}(4 \cdot \omega_c) = P_{LR}(0) \qquad P_{LR}(3 \cdot \omega_c) = P_{LR}(-\omega_c) \qquad P_{LR}(5 \cdot \omega_c) = P_{LR}(\omega_c)$$

Example

- Low-pass filter 4th order, 4 GHz cutoff frequency, maximally flat design (working with 50Ω source and load)
- maximally flat table or formulas:
  - $g_1 = 0.7654 = L_1$
  - $g_2 = 1.8478 = C_2$
  - $g_3 = 1.8478 = L_3$
  - $g_4 = 0.7654 = C_4$
  - $g_5 = 1$ (**does not** need supplemental impedance matching – required only for even order equal-ripple filters)

LPF Prototype





Lumped elements

$$\omega_c = 2 \cdot \pi \cdot 4\,GHz = 2.5133 \cdot 10^{10}\,rad/s$$

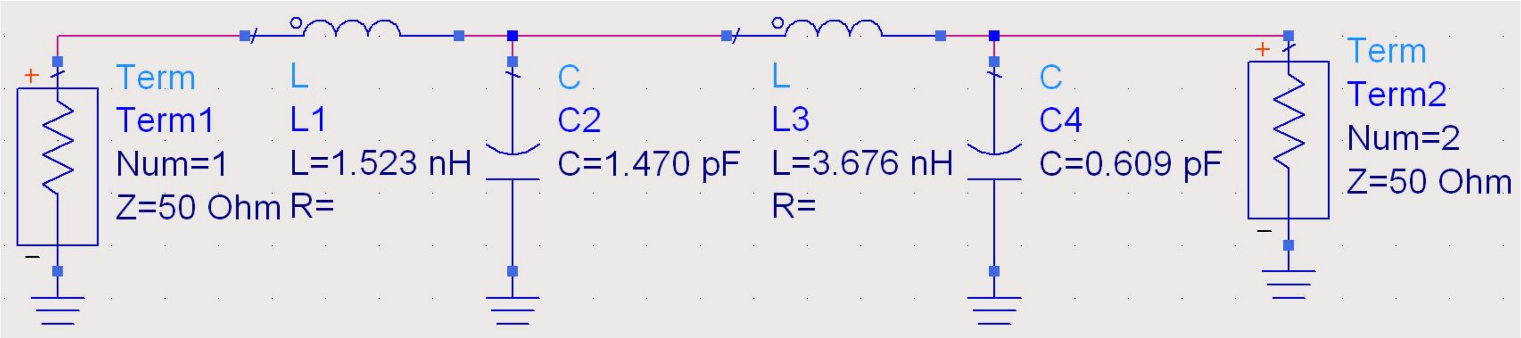
g1 = 0.7654 = L1,
g2 = 1.8478 = C2,

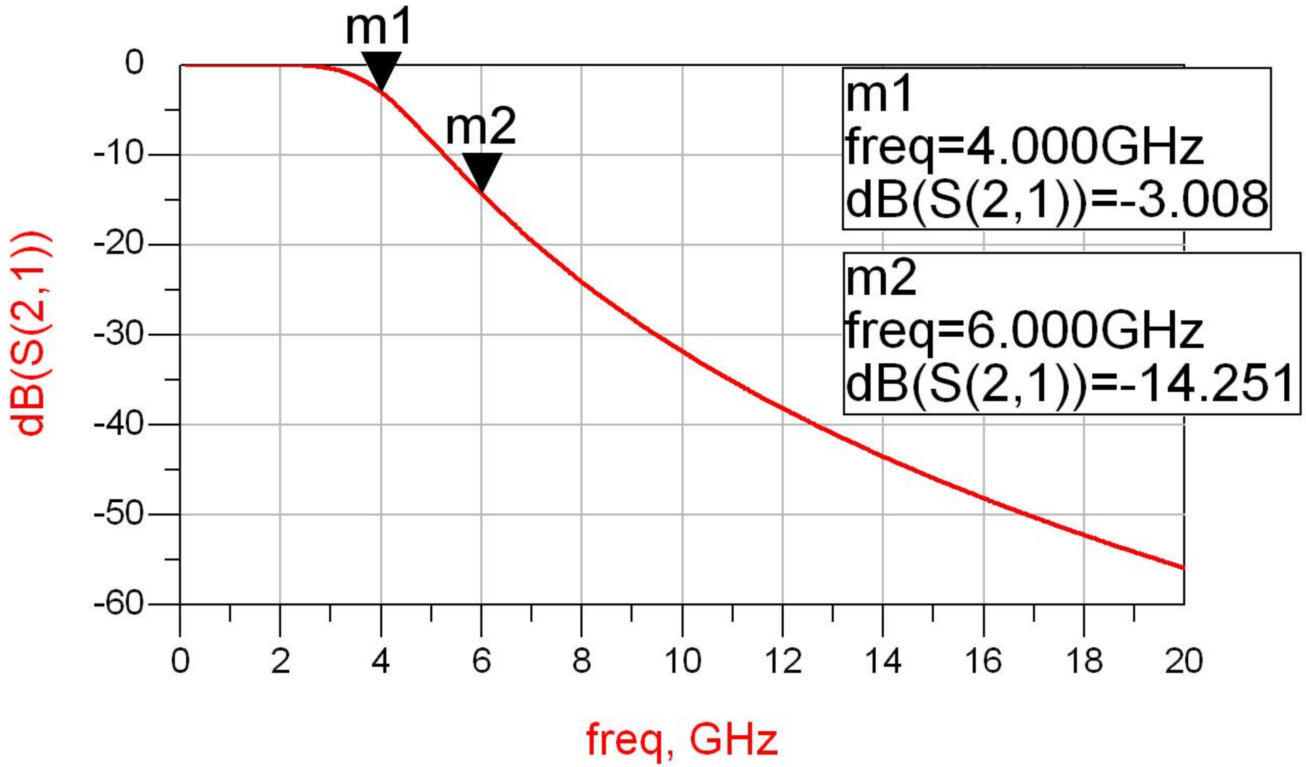
g3 = 1.8478 = L3,
g4 = 0.7654 = C4,
g5 = 1 = RL

$$L_1' = \frac{R_0 \cdot L_1}{\omega_c} = 1.523\,nH \qquad C_2' = \frac{C_2}{R_0 \cdot \omega_c} = 1.470\,pF$$

$$L_3' = \frac{R_0 \cdot L_3}{\omega_c} = 3.676\,nH \qquad C_4' = \frac{C_4}{R_0 \cdot \omega_c} = 0.609\,pF$$

Lumped elements – ADS





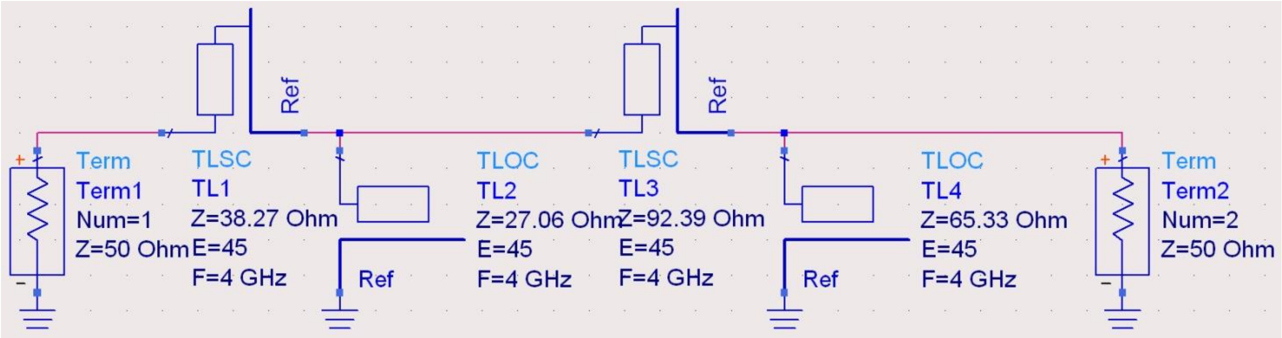
Richards' Transformation

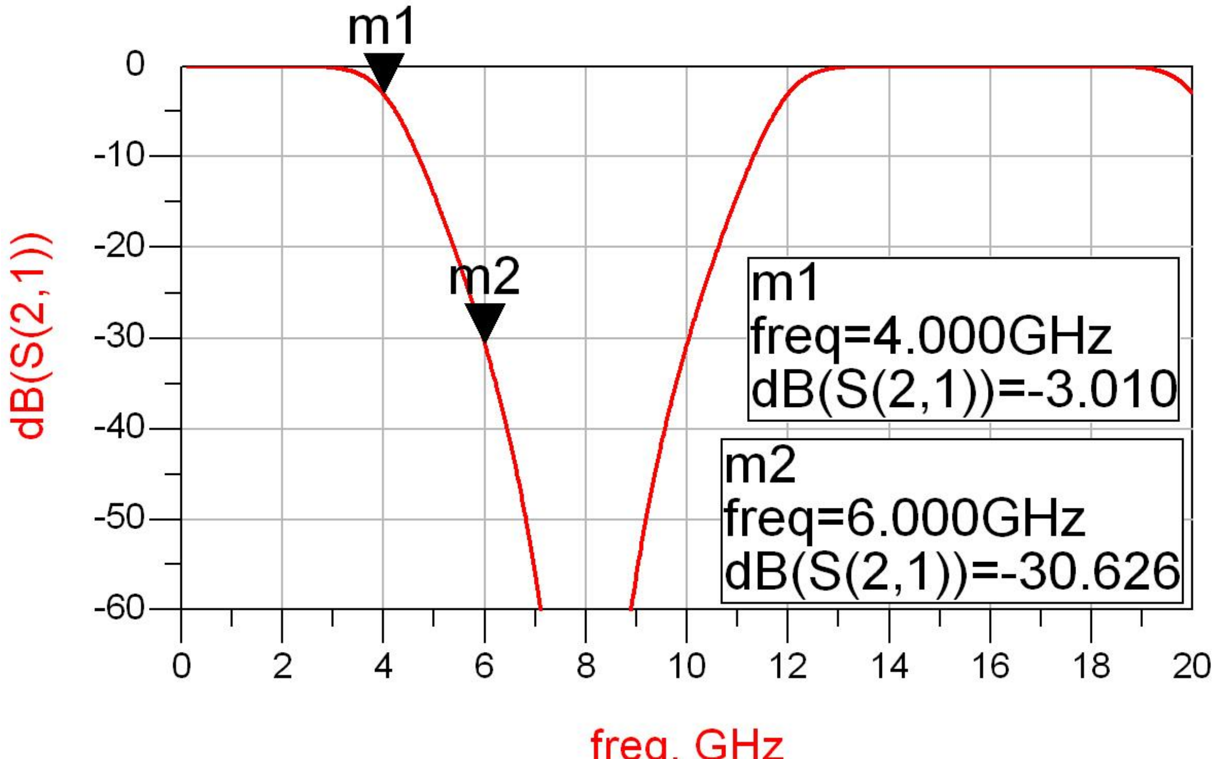
- LPF Prototype parameters:
  - g1 = 0.7654 = L1
  - g2 = 1.8478 = C2
  - g3 = 1.8478 = L3
  - g4 = 0.7654 = C4
- Normalized line impedances
  - z1 = 0.7654 = series / short circuit
  - z2 = 1 / 1.8478 = 0.5412 = shunt / open circuit
  - z3 = 1.8478 = series / short circuit
  - z4 = 1/ 0.7654 = 1.3065 = shunt / open circuit
- Impedance scaling by multiplying with Zo = 50Ω
- All lines must have the length equal to λ/8 (electrical length E = 45°) at 4GHz

$$Z_0 \leftrightarrow \frac{1}{C}$$

$$Z_0 \leftrightarrow L$$

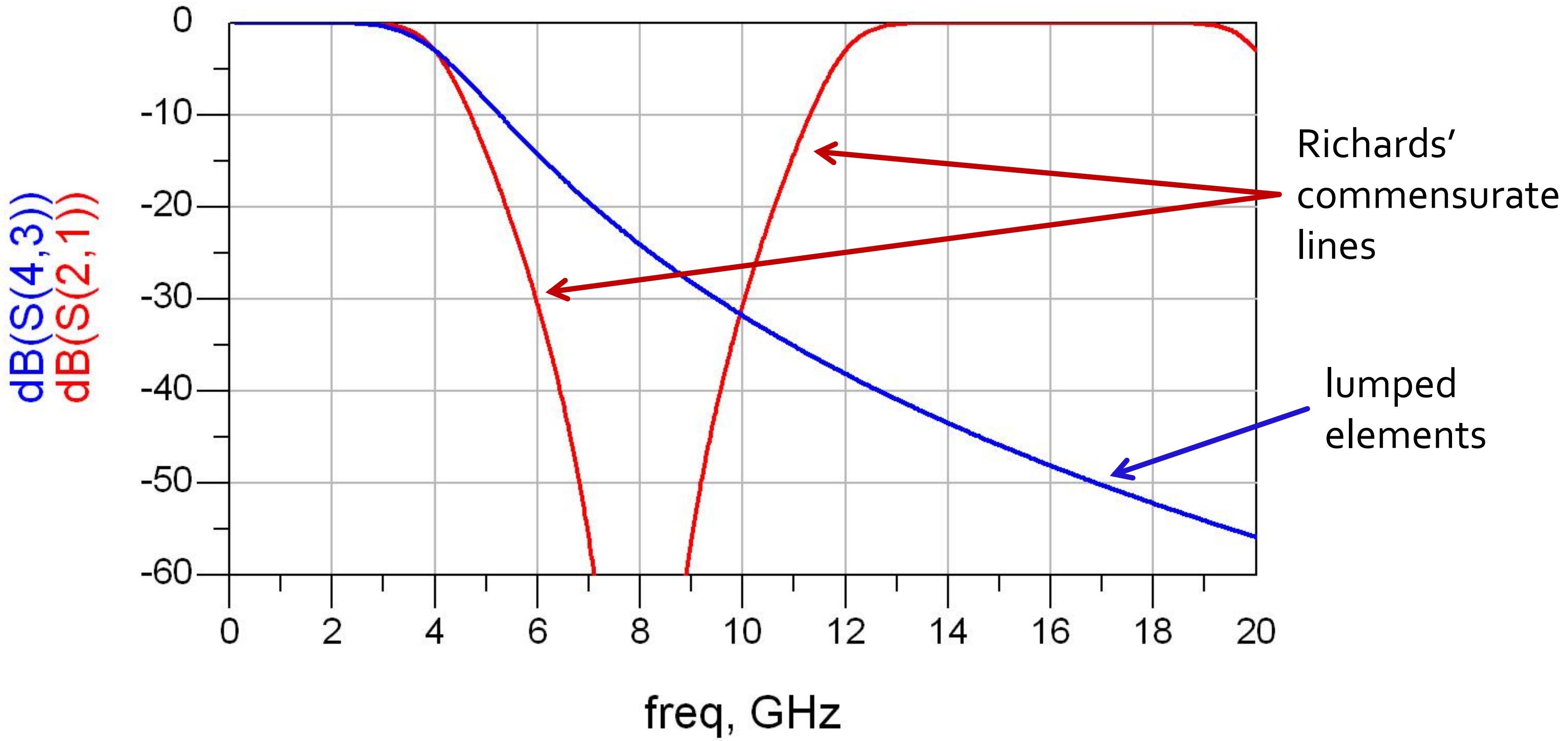
Richards' Transformation – ADS





Richards' Transformation

- Filters implemented with Richards' Transformation
  - beneficiate from the supplemental pole at $2 \cdot \omega_c$
  - have the major disadvantage of frequency periodicity, a supplemental non-periodic LPF must be inserted if needed



Equal-ripple prototype

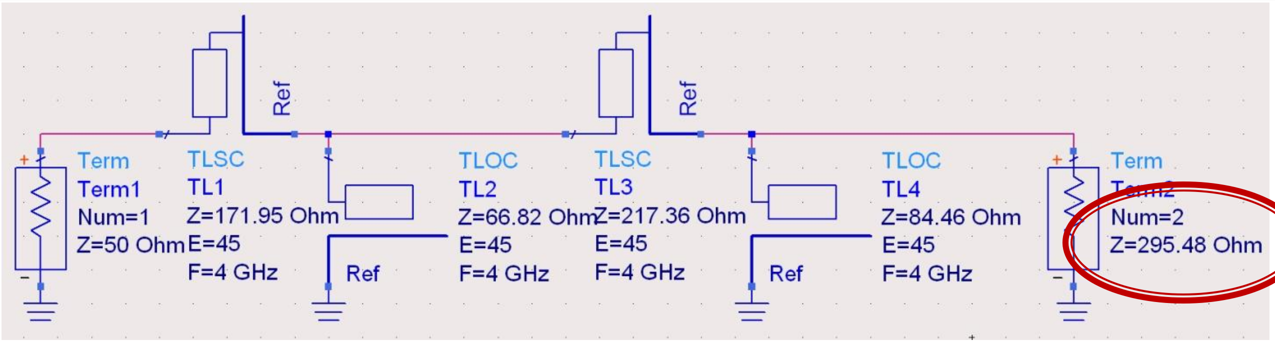
- For even N order of the filter (N = 2, 4, 6, 8 …) equal-ripple filters **must** closed by a non-standard load impedance $\mathbf{g_{N+1} \neq 1}$
- If the application doesn't allow this, supplemental impedance matching is required (quarter-wave transformer, binomial …) to $\mathbf{g_L = 1}$

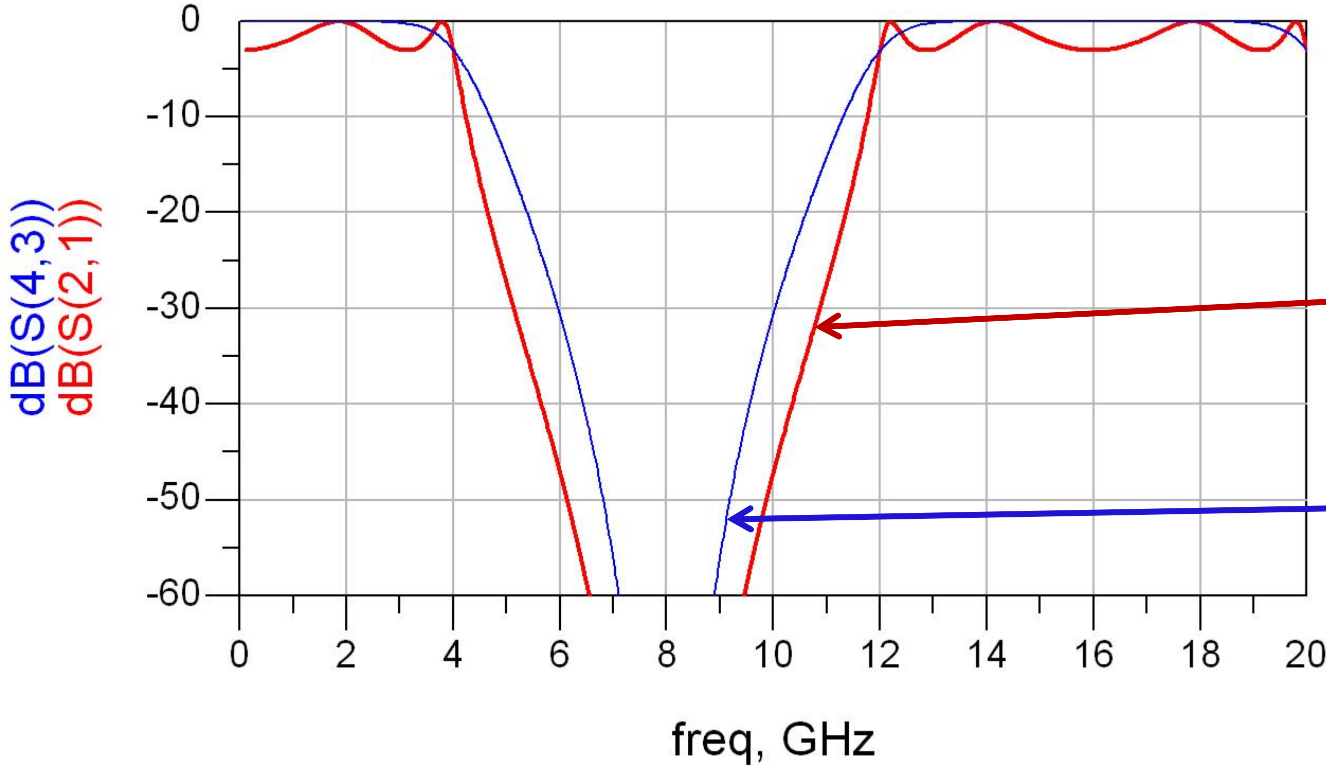
$$g_{N+1} \neq 1 \rightarrow R \neq R_0 \quad (50\Omega)$$

Observation: even order equal-ripple

- Same filter, 3dB equal-ripple
- 3dB equal-ripple tables or formulas:
  - g1 = 3.4389 = L1
  - g2 = 0.7483 = C2
  - g3 = 4.3471 = L3
  - g4 = 0.5920 = C4
  - g5 = 5.8095 = R_L
- Line impedances
  - Z1 = 3.4389·50Ω = 171.945Ω = series / short circuit
  - Z2 = 50Ω / 0.7483 = 66.818Ω = shunt / open circuit
  - Z3 = 4.3471·50Ω = 217.355Ω = series / short circuit
  - Z4 = 50Ω / 0.5920 = 84.459Ω = shunt / open circuit
  - RL = 5.8095·50Ω = 295.475Ω = load

Even order equal-ripple – ADS



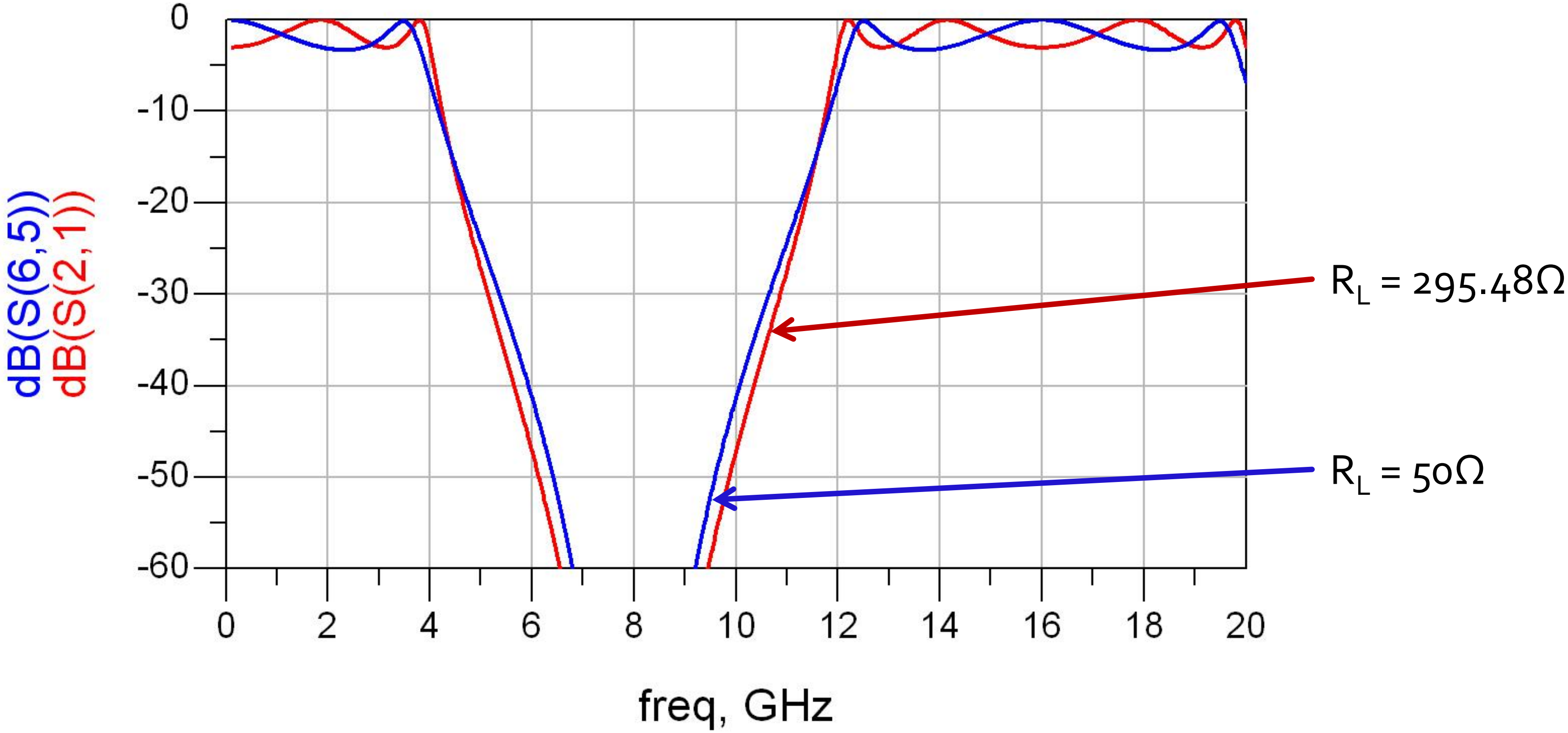


3dB equal-ripple (4^{th} ord.)

maximum flat (4^{th} ord)

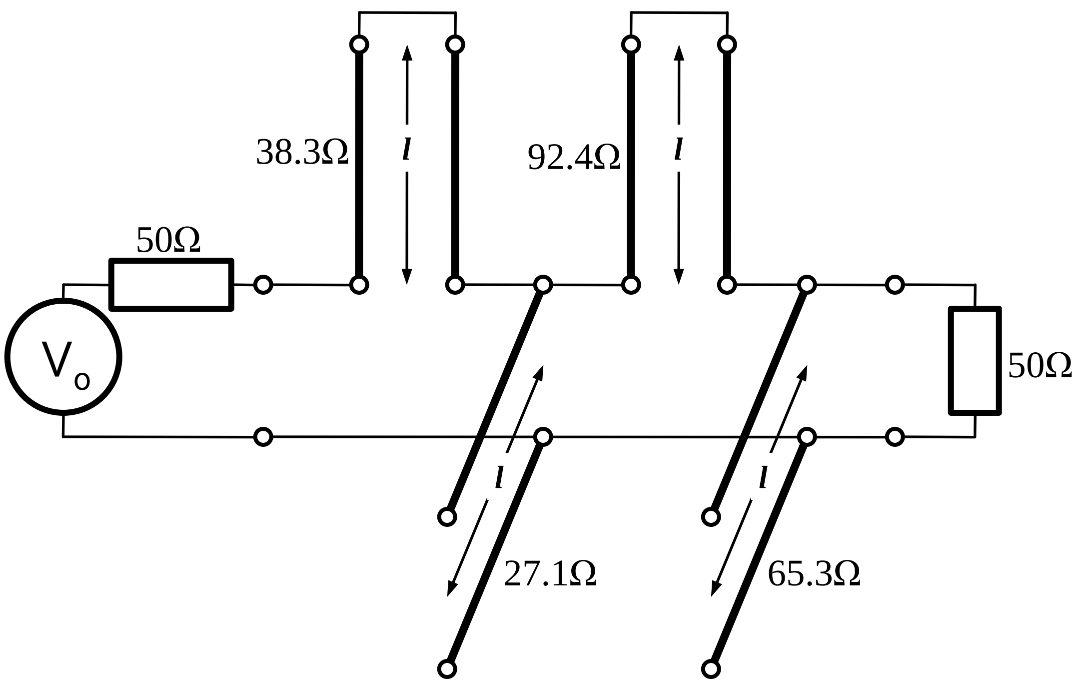
Observation: even order equal-ripple

- Even order equal-ripple filters need output matching towards 50Ω for precise results. Example:



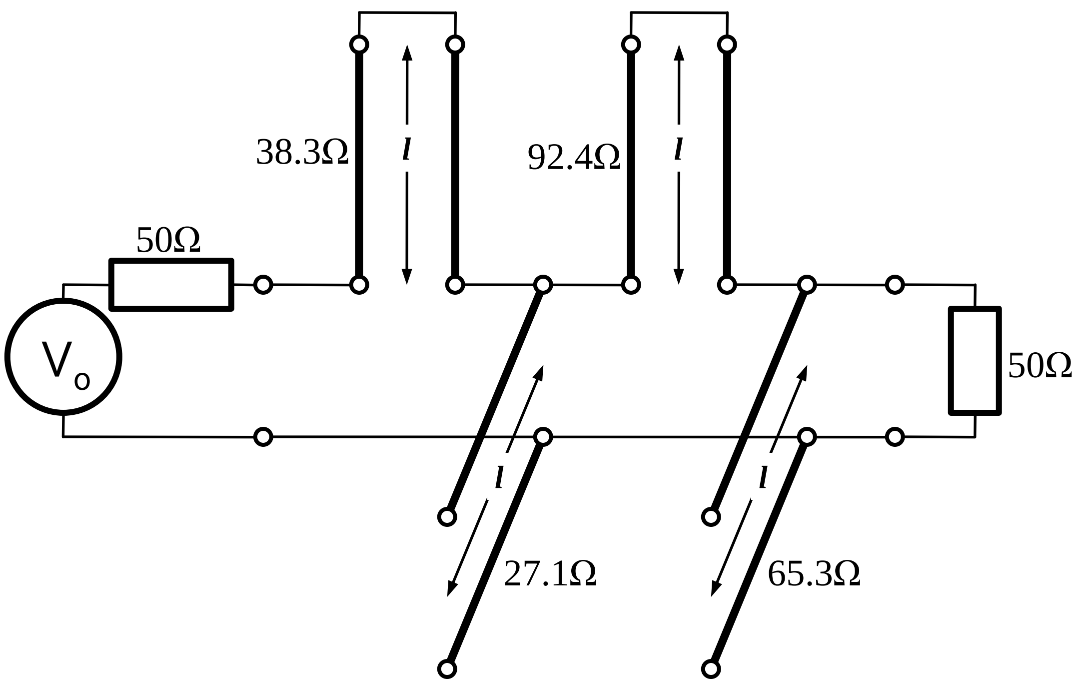
Kuroda's Identities

- Filters implemented with the Richards' transformation have certain disadvantages in terms of practical use
- Kuroda's Identities/ Transformations can eliminate some of these disadvantages
- We use additional line sections to obtain systems that are easier to implement in practice
- The additional line sections are called unit elements and have lengths of λ / 8 at the desired cutoff frequency (ω_c) thus being commensurate with the stubs implementing the inductors and capacitors.



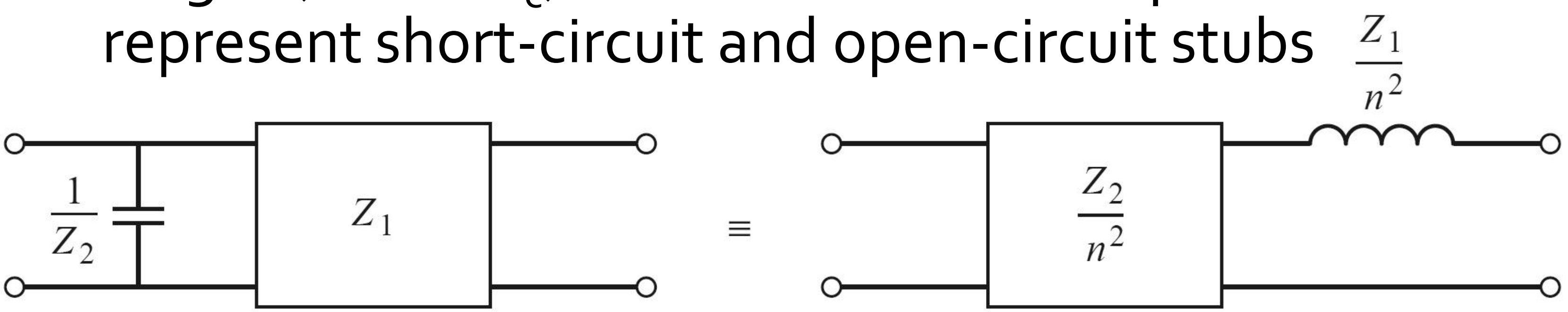
Kuroda's Identities

- Kuroda's Identities perform any of the following operations:
  - Physically separate transmission line stubs
  - Transform series stubs into shunt stubs, or vice versa
  - Change impractical characteristic impedances into more realizable values (~50Ω)

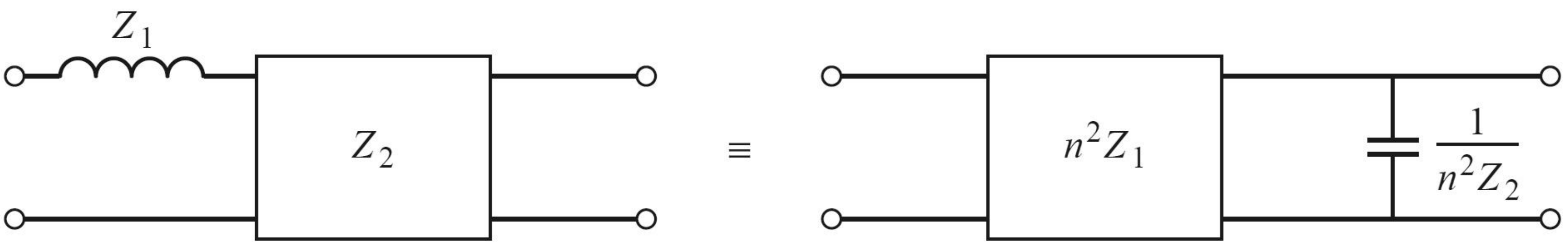


Kuroda's Identities

- 4 circuit equivalents (a,b)
  - each box represents a unit element, or transmission line, of the indicated characteristic impedance and length ($\lambda/8$ at ω_c). The inductors and capacitors represent short-circuit and open-circuit stubs



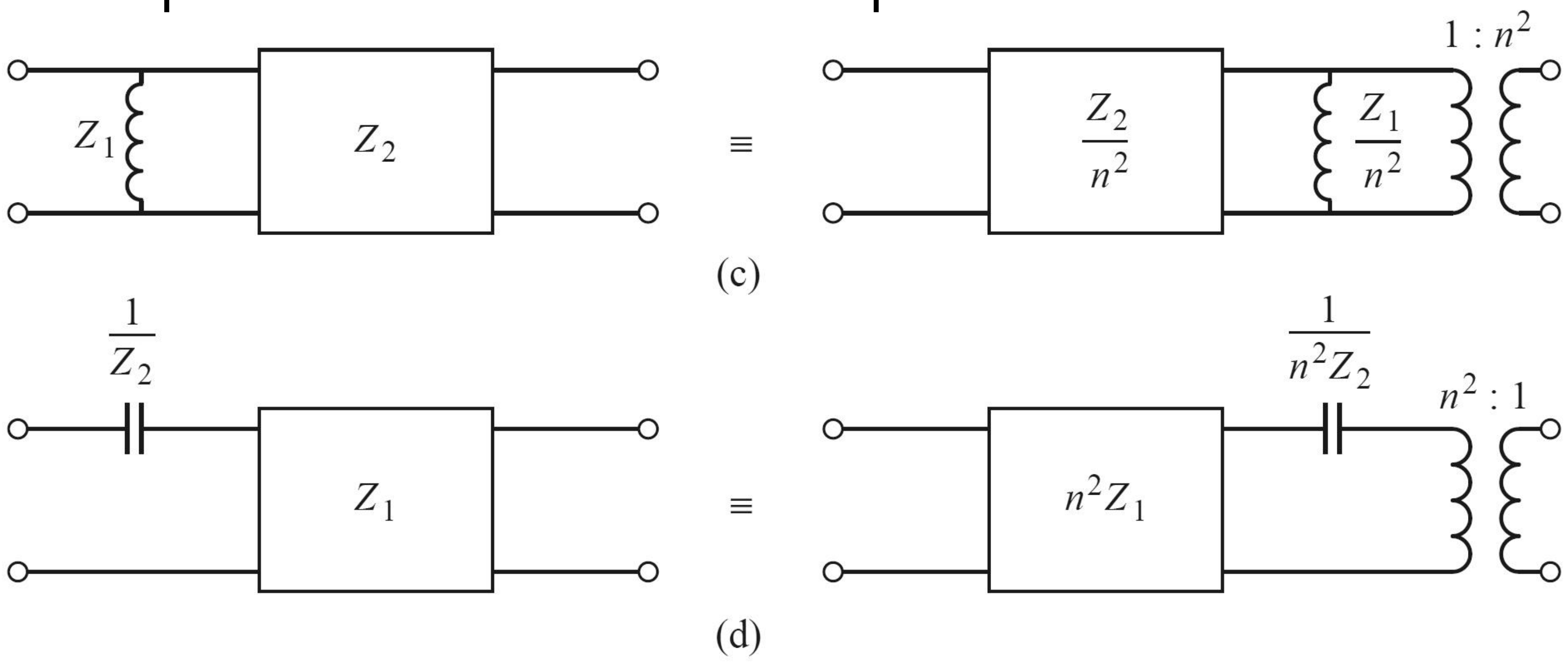
(a)



(b)

Kuroda's Identities

- 4 circuit equivalents (c,d)
  - each box represents a unit element, or transmission line, of the indicated characteristic impedance and length ($\lambda/8$ at ω_c). The inductors and capacitors represent short-circuit and open-circuit stubs



Kuroda's Identities

- In all Kuroda's Identities:
  - n:

$$n^2 = 1 + \frac{Z_2}{Z_1}$$

  - The inductors and capacitors represent short-circuit and open-circuit stubs resulted from Richards' transformation ($\lambda/8$ at ω_c).
  - Each box represents a unit element, or transmission line, of the indicated characteristic impedance and length ($\lambda/8$ at ω_c).

First Kuroda's Identity

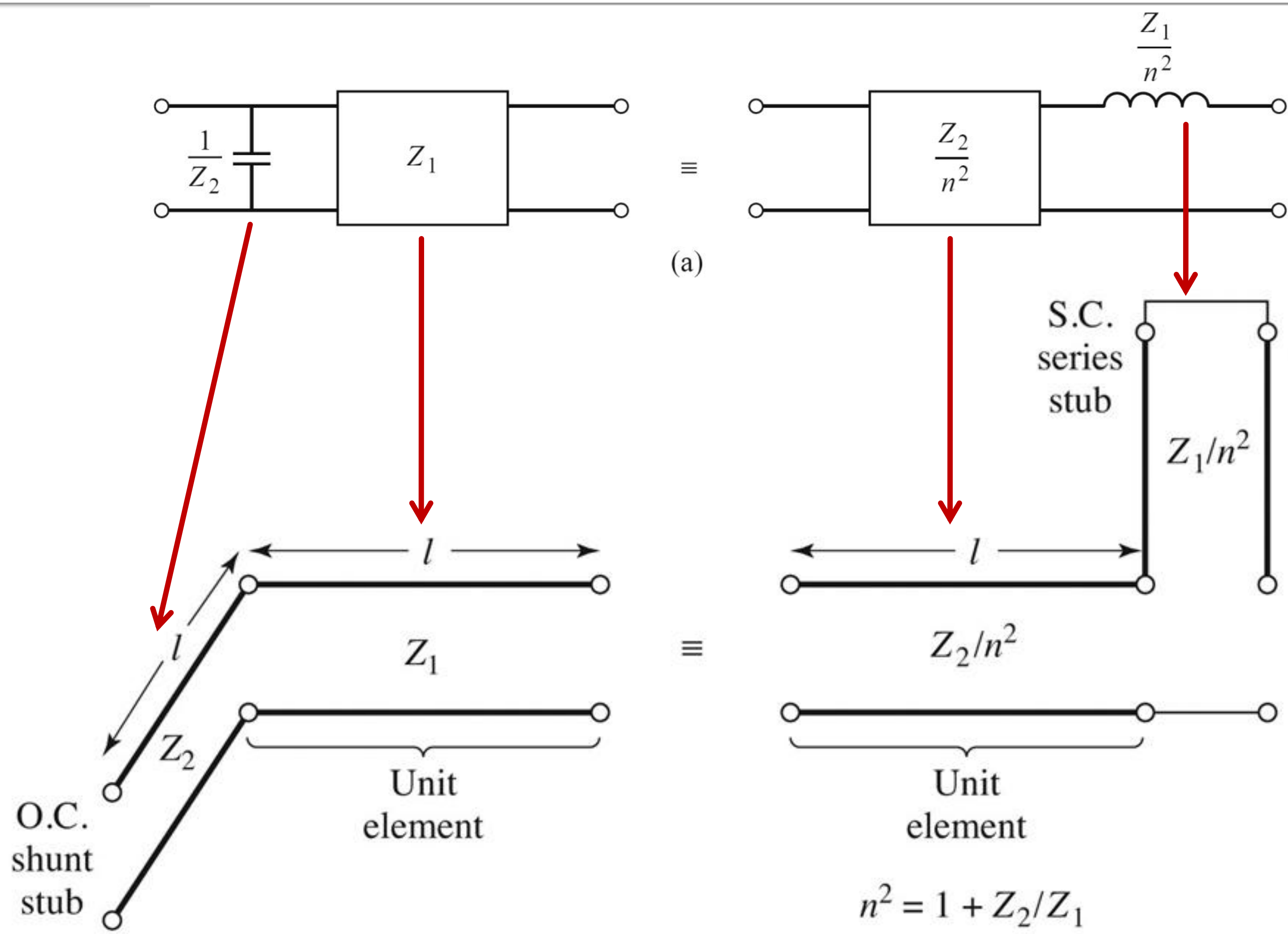
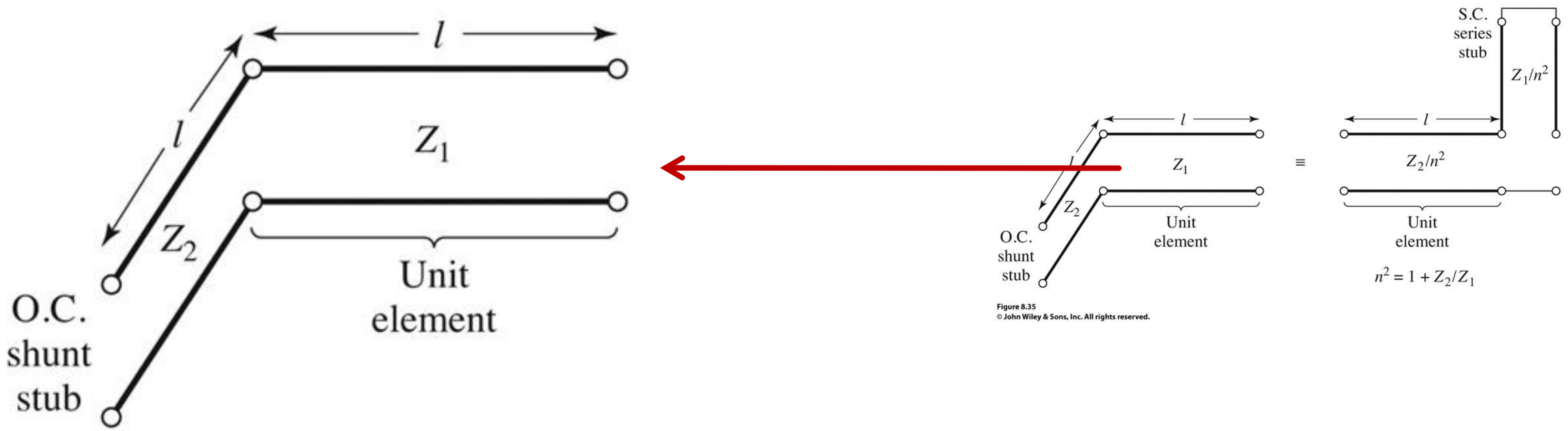
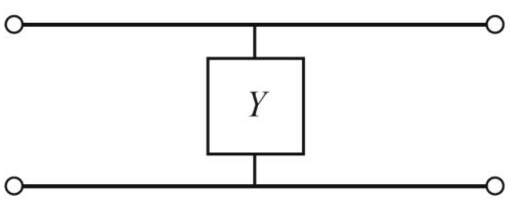


Figure 8.35

First Kuroda's Identity – Proof



- ABCD matrices, L4



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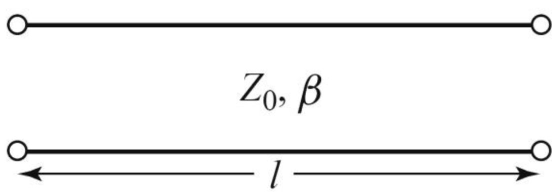
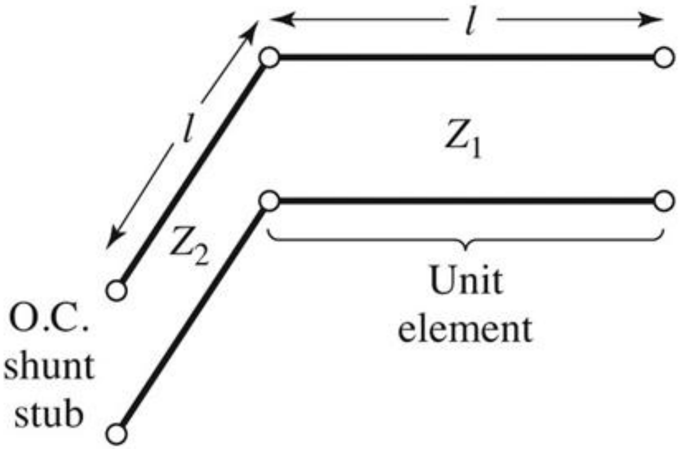


$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta\cdot l & j\cdot Z_0\cdot\sin\beta\cdot l \\ j\cdot Y_0\cdot\sin\beta\cdot l & \cos\beta\cdot l \end{bmatrix}$$

First Kuroda's Identity – Proof



$$\Omega = \tan \beta \cdot l$$

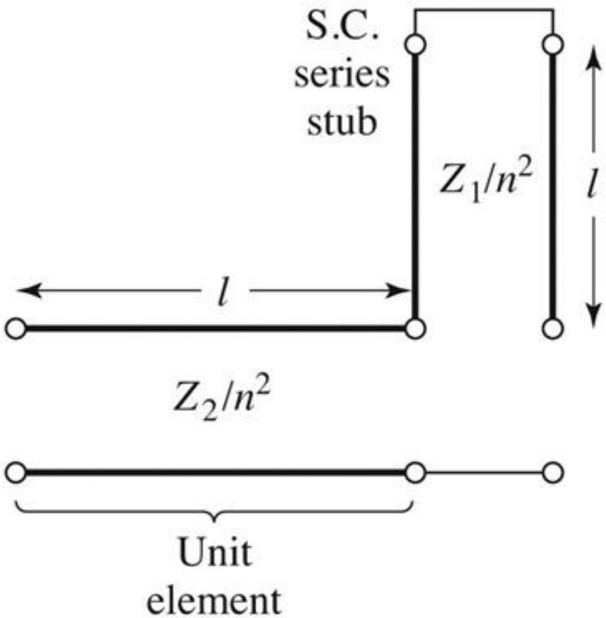
$$\cos \beta \cdot l = \frac{1}{\sqrt{1+\Omega^2}} \qquad \sin \beta \cdot l = \frac{\Omega}{\sqrt{1+\Omega^2}}$$

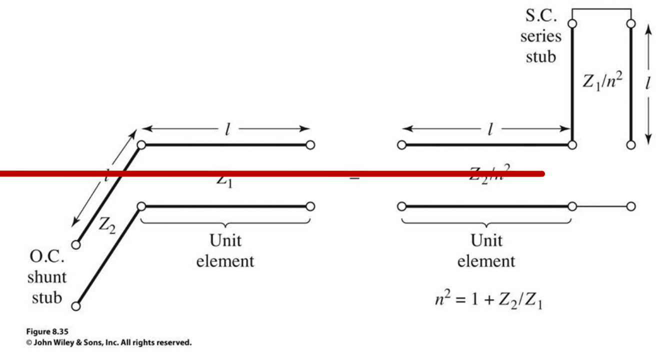
$$Z_{in,oc} = -j \cdot Z_2 \cdot \cot \beta \cdot l = -j \cdot \frac{Z_2}{\Omega}$$

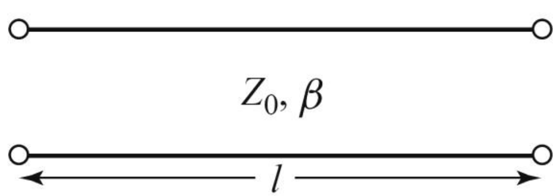
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \dfrac{j \cdot \Omega}{Z_2} & 1 \end{bmatrix} \cdot \begin{bmatrix} \dfrac{1}{\sqrt{1+\Omega^2}} & j \cdot Z_1 \cdot \dfrac{\Omega}{\sqrt{1+\Omega^2}} \\ j \cdot \dfrac{1}{Z_1} \cdot \dfrac{\Omega}{\sqrt{1+\Omega^2}} & \dfrac{1}{\sqrt{1+\Omega^2}} \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}} \cdot \begin{bmatrix} 1 & 0 \\ \dfrac{j \cdot \Omega}{Z_2} & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & j \cdot \Omega \cdot Z_1 \\ \dfrac{j \cdot \Omega}{Z_1} & 1 \end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}} \cdot \begin{bmatrix} 1 & j \cdot \Omega \cdot Z_1 \\ j \cdot \Omega \cdot \left(\dfrac{1}{Z_1} + \dfrac{1}{Z_2} \right) & 1 - \Omega^2 \cdot \dfrac{Z_1}{Z_2} \end{bmatrix}$$

First Kuroda's Identity – Proof







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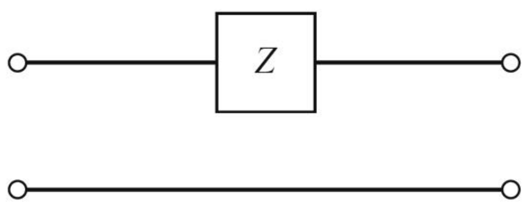
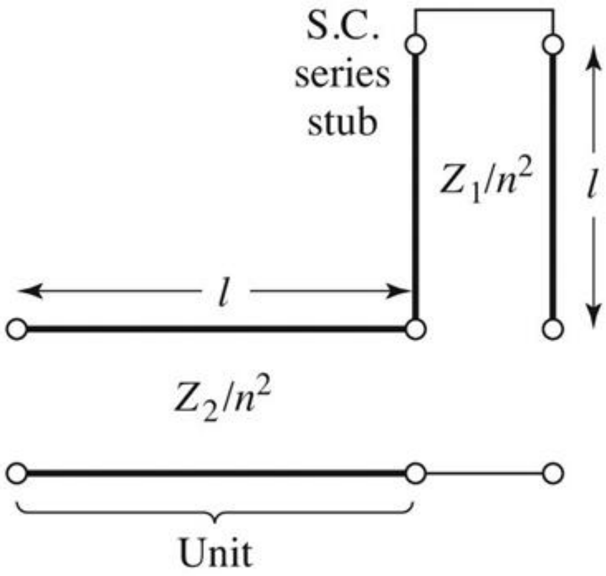


$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta\cdot l & j\cdot Z_0\cdot\sin\beta\cdot l \\ j\cdot Y_0\cdot\sin\beta\cdot l & \cos\beta\cdot l \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

First Kuroda's Identity – Proof



$$\Omega = \tan\beta\cdot l$$

$$\cos\beta\cdot l = \frac{1}{\sqrt{1+\Omega^2}} \qquad \sin\beta\cdot l = \frac{\Omega}{\sqrt{1+\Omega^2}}$$

$$Z_{in,sc} = j\cdot\left(\frac{Z_1}{n^2}\right)\cdot\tan\beta\cdot l = \frac{j\cdot\Omega\cdot Z_1}{n^2}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{1+\Omega^2}} & j\cdot\frac{Z_2}{n^2}\cdot\frac{\Omega}{\sqrt{1+\Omega^2}} \\ j\cdot\frac{n^2}{Z_2}\cdot\frac{\Omega}{\sqrt{1+\Omega^2}} & \frac{1}{\sqrt{1+\Omega^2}} \end{bmatrix}\cdot\begin{bmatrix} 1 & \frac{j\cdot\Omega\cdot Z_1}{n^2} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}}\cdot\begin{bmatrix} 1 & j\cdot\Omega\cdot\frac{Z_2}{n^2} \\ \frac{j\cdot\Omega\cdot n^2}{Z_2} & 1 \end{bmatrix}\cdot\begin{bmatrix} 1 & j\cdot\Omega\cdot\frac{Z_1}{n^2} \\ 0 & 1 \end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}}\cdot\begin{bmatrix} 1 & j\cdot\frac{\Omega}{n^2}\cdot(Z_1+Z_2) \\ \frac{j\cdot\Omega\cdot n^2}{Z_2} & 1-\Omega^2\cdot\frac{Z_1}{Z_2} \end{bmatrix}$$

First Kuroda's Identity – Proof

- First circuit

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}} \cdot \begin{bmatrix} 1 & j \cdot \Omega \cdot Z_1 \\ j \cdot \Omega \cdot \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right) & 1 - \Omega^2 \cdot \frac{Z_1}{Z_2} \end{bmatrix}$$

- Second circuit

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}} \cdot \begin{bmatrix} 1 & j \cdot \frac{\Omega}{n^2} \cdot (Z_1 + Z_2) \\ \frac{j \cdot \Omega \cdot n^2}{Z_2} & 1 - \Omega^2 \cdot \frac{Z_1}{Z_2} \end{bmatrix}$$

- Results are identical if we choose

$$n^2 = 1 + \frac{Z_2}{Z_1}$$

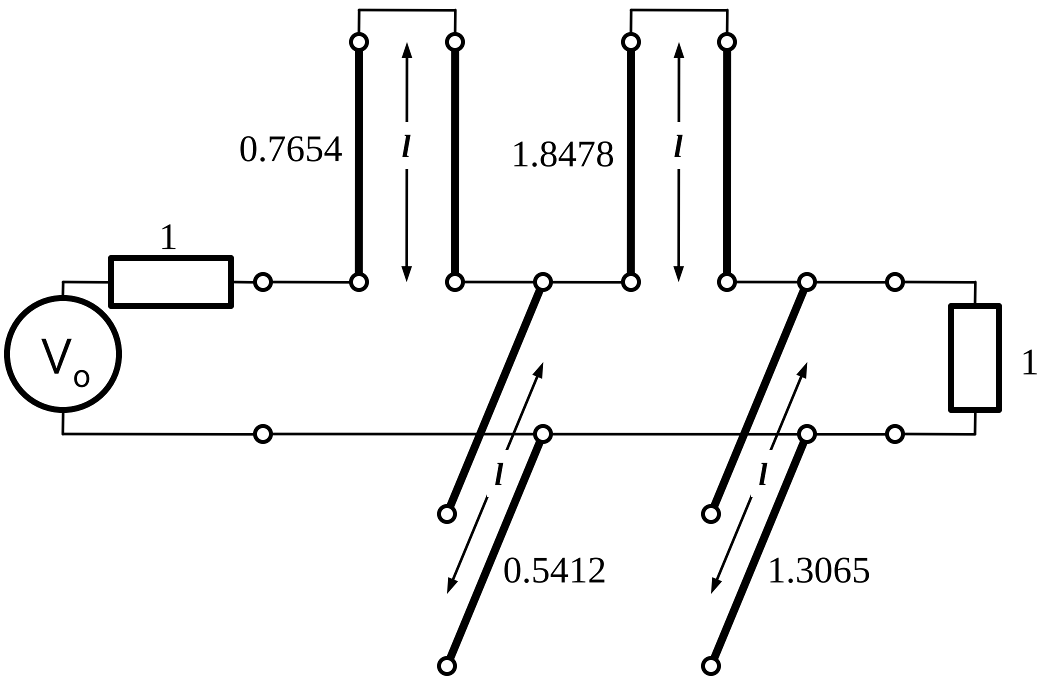
- The other 3 identities can be proved in the same way

(Same) Example

- Low-pass filter 4^{th} order, 4 GHz cutoff frequency, maximally flat design (working with 50Ω source and load)
- maximally flat table or formulas:
  - $g1 = 0.7654 = L1$
  - $g2 = 1.8478 = C2$
  - $g3 = 1.8478 = L3$
  - $g4 = 0.7654 = C4$
  - $g5 = 1$ (**does not** need supplemental impedance matching – required only for even order equal-ripple filters)

Example

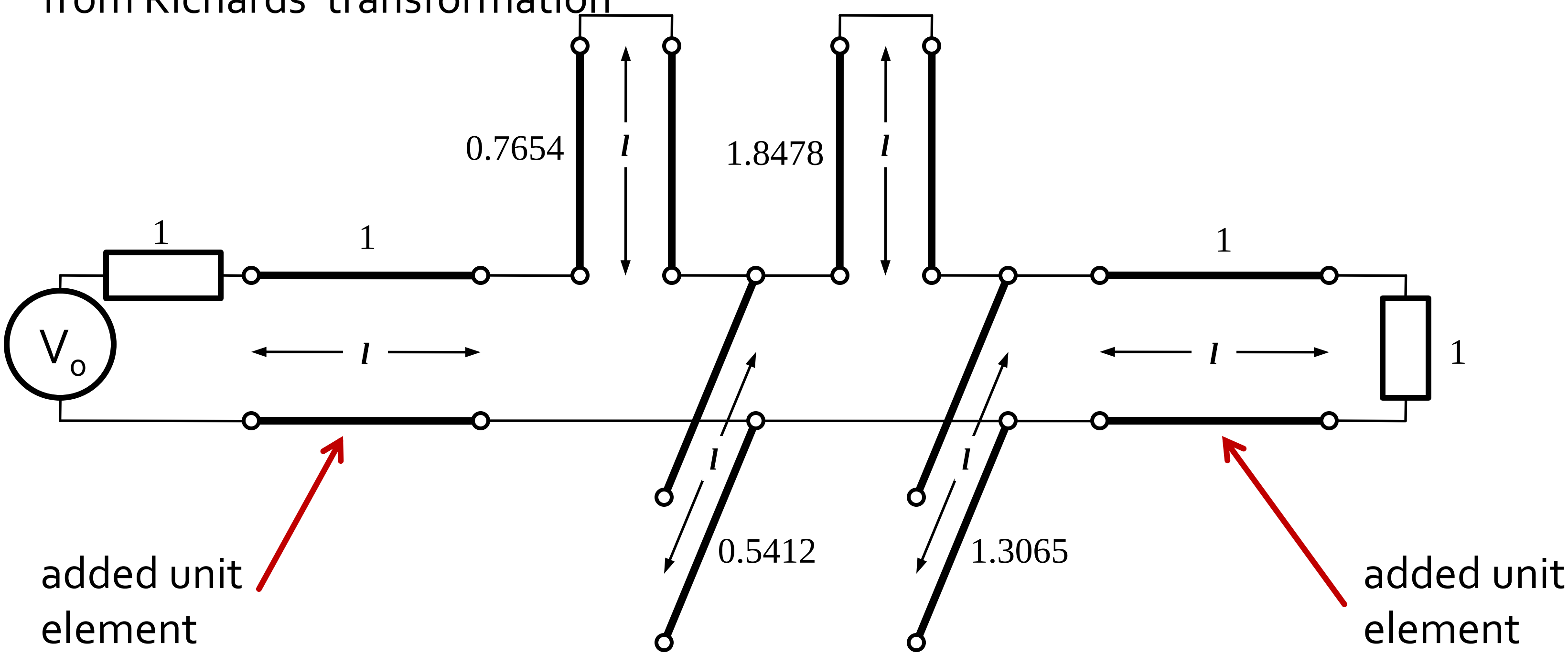
- Apply Richards's transformation



- Problems:
  - the series stubs would be **very difficult** to implement in microstrip line form
  - in microstrip technology it is preferable to have open-circuit stubs (short-circuit requires a **via-hole** to the ground plane)
  - the 4 stubs are physically connected at the same point, an implementation that eliminates/reduces the **coupling** between these lines is impossible
  - not the case here, but sometimes the normalized impedances are much different from 1. Most circuit technologies are designed for 50Ω lines

Example

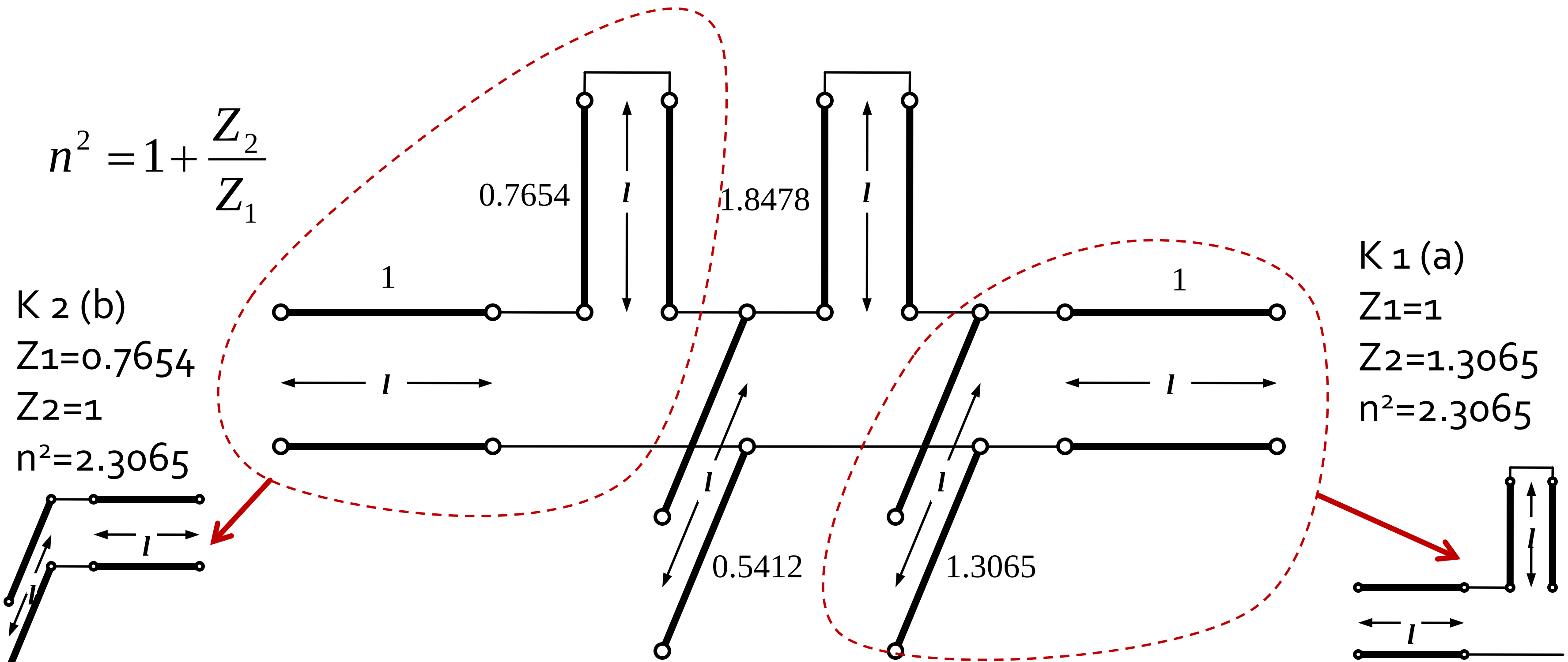
- In all 4 Kuroda's Identities we **always** have a circuit with a series line section (not present in initial circuit):
  - we **add** unit elements (z = 1, l = λ/8) at the ends of the filter (these redundant elements do not affect filter performance since they are matched to z = 1, both source and load)
  - we **apply** one of the Kuroda's Identities at both ends and **continue** (add unit …)
  - we can **stop** the procedure when we have a series line section between all the stubs from Richards' transformation



Example

- Apply:
  - Kuroda 2 (L,Z known → C,Z) on the left side
  - Kuroda 1 (C,Z known → L,Z) on the right side

$$n^2 = 1 + \frac{Z_2}{Z_1}$$

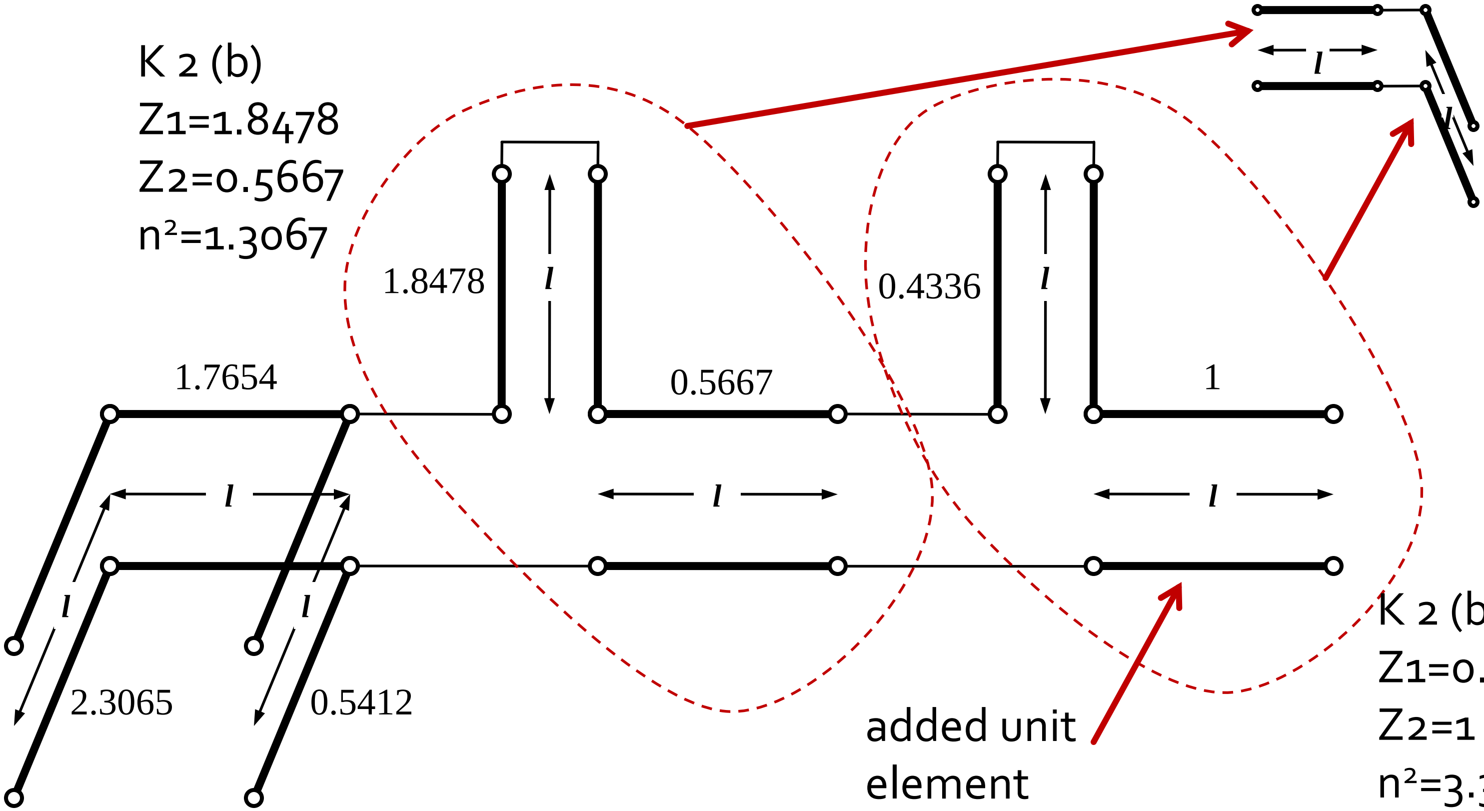


Example

- We add another unit element on the right side and apply Kuroda 2 twice

$$n^2 = 1 + \frac{Z_2}{Z_1}$$

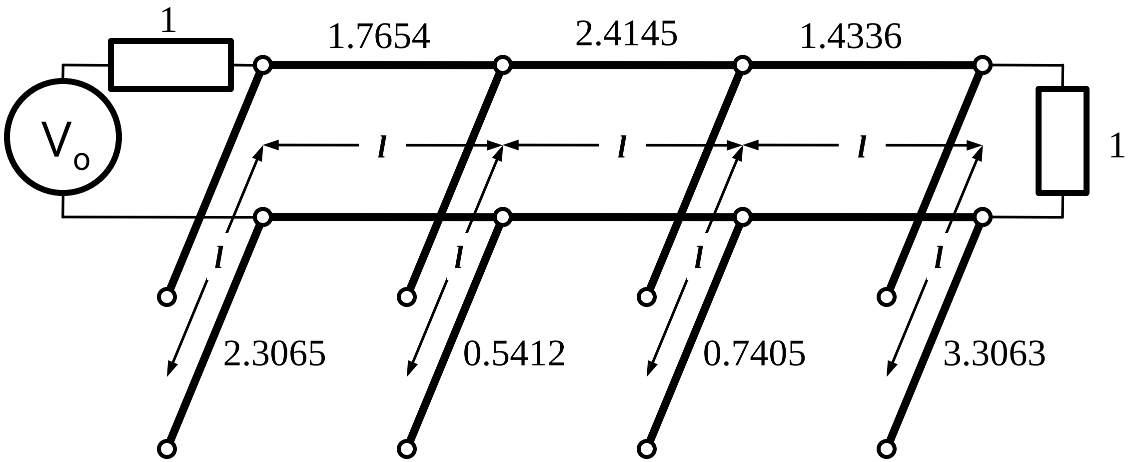
K 2 (b)
Z1=1.8478
Z2=0.5667
n^2=1.3067



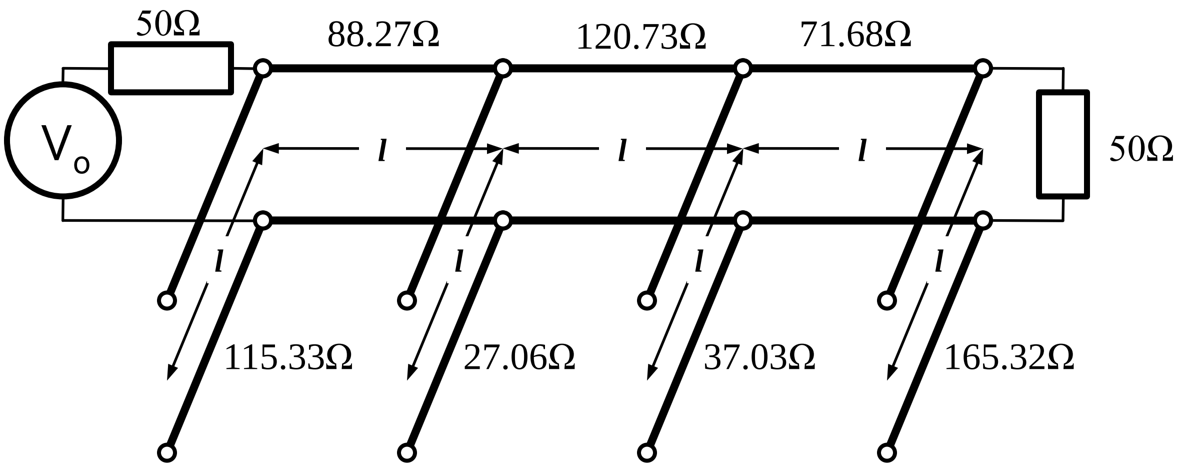
added unit element

K 2 (b)
Z1=0.4336
Z2=1
n^2=3.3063

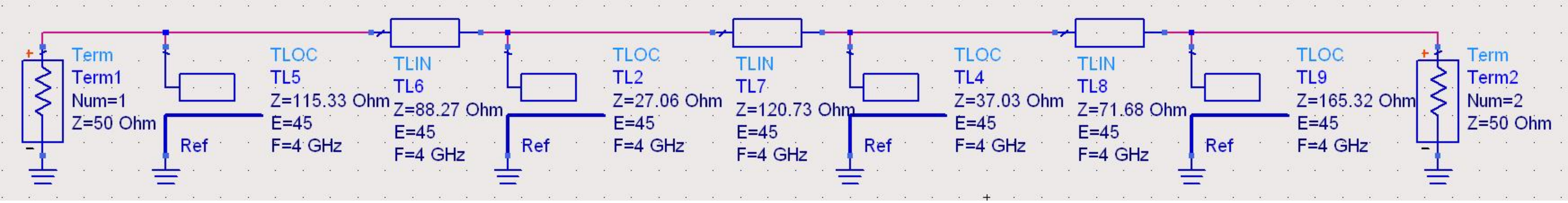
Example

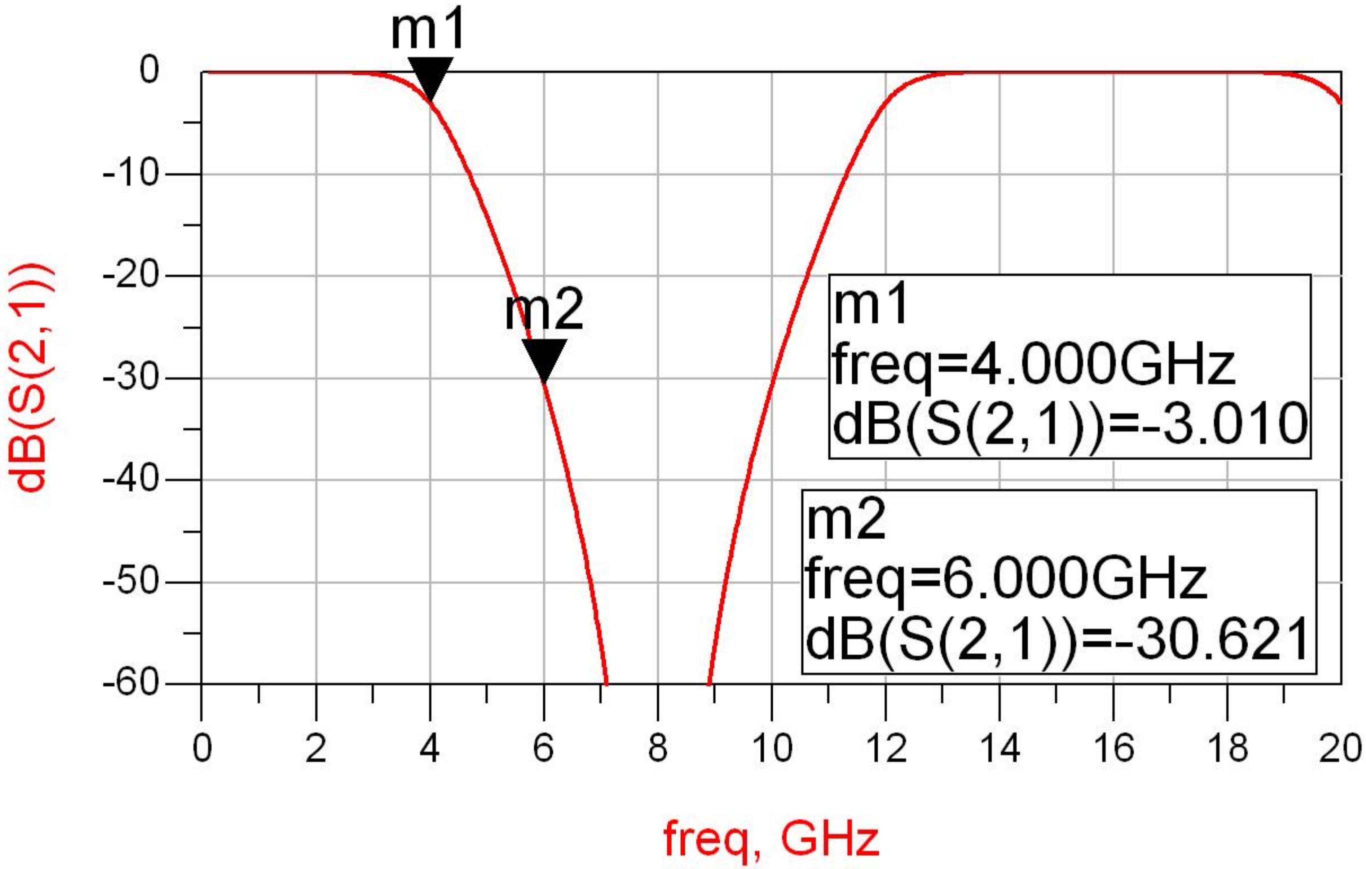


- Impedance scaling (multiply by 50Ω)



Kuroda's Identities – ADS





Examples

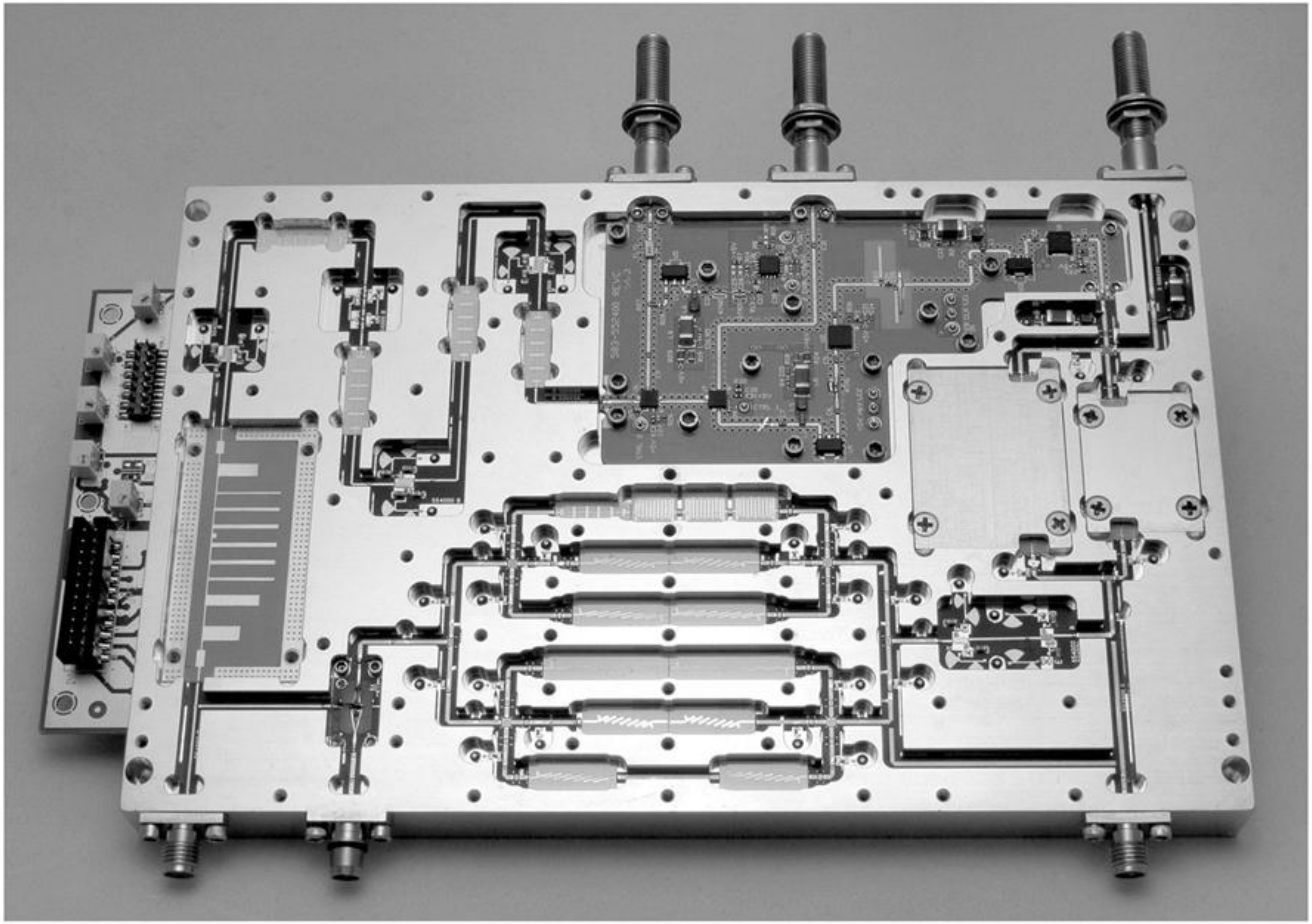


Figure 8.55
Courtesy of LNX Corporation, Salem, N.H.

Examples

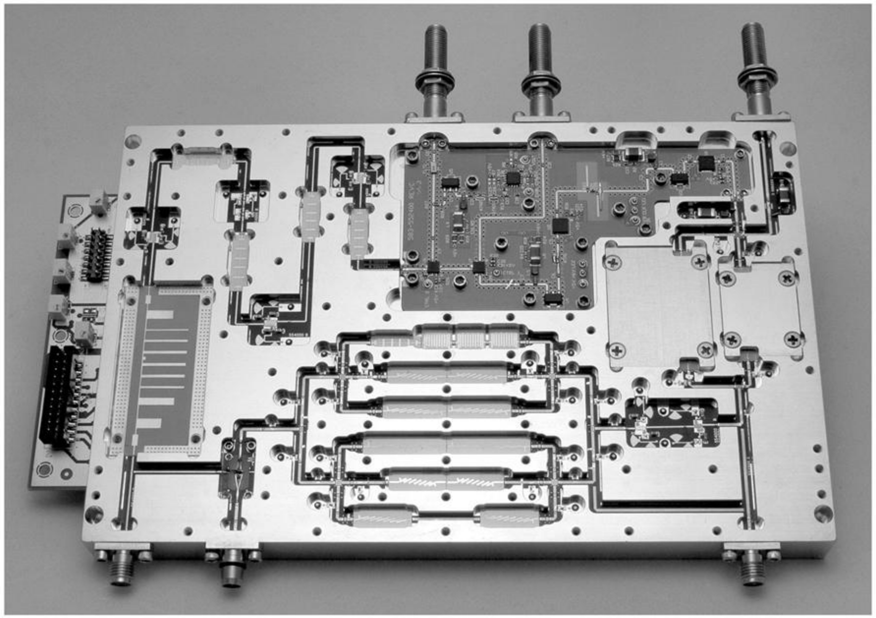
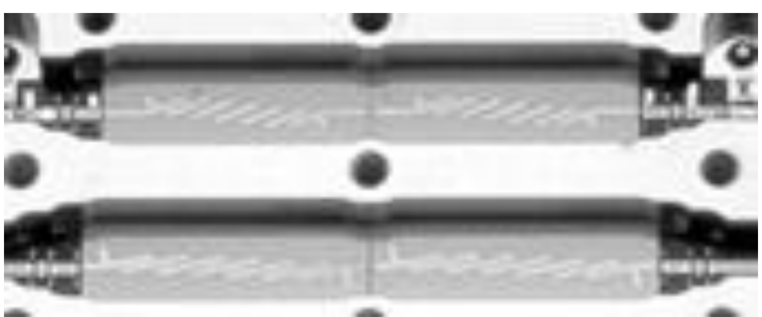
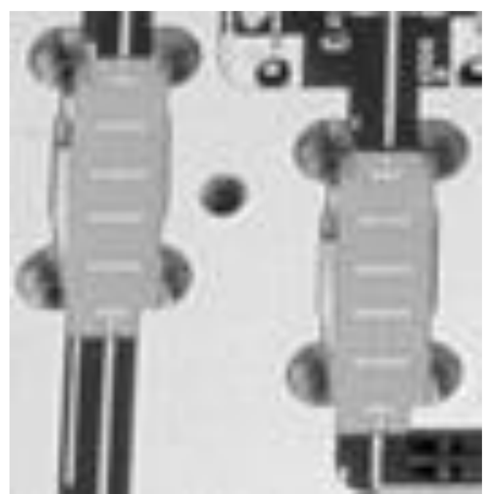
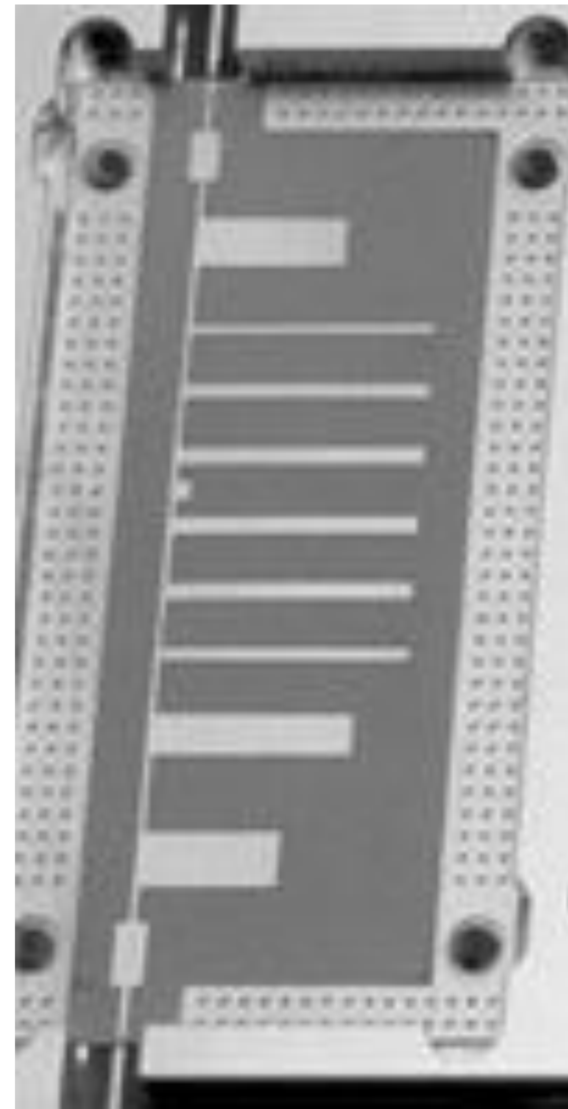


Figure 8.55
Courtesy of LNX Corporation, Salem, N.H.







Contact

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